Method of Partial Fractions

I.) Distinct Linear Factors

Ex: $\int \frac{x-3}{x^2+2x-3}\,dx = \int \frac{x-3}{(x-1)(x+3)}\,dx$

$= \int \left[\frac{A}{x-1} + \frac{B}{x+3}\right] dx$

Ex: $\int \frac{x^2+x+1}{x^3-4x}\,dx = \int \frac{x^2+x+1}{x(x^2-4)}\,dx$

$= \int \frac{x^2+x+1}{x(x-2)(x+2)}\,dx = \int \left[\frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+2}\right] dx$

II.) Repeated Linear Factors

Ex: $\int \frac{4-3x}{x^2(x+2)}\,dx = \int \left[\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+2}\right] dx$

Ex: $\int \frac{x^3+7}{x(x-1)^3}\,dx = \int \left[\frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2} + \frac{D}{(x-1)^3}\right] dx$

III.) Irreducible Quadratic

Ex: $\int \frac{3x-5}{x(x^2+1)}\,dx = \int \left[\frac{A}{x} + \frac{Bx+C}{x^2+1}\right] dx$

IV. Polynomial Division First

(If the degree of the numerator is equal to or larger than the degree of the denominator)

<u>Ex</u>: $\int \frac{x^3+x^2-3}{x^2-1}\,dx$

$$= \int \left[x+1+\frac{x-2}{x^2-1}\right]dx$$

$$= \frac{1}{2}x^2+x+\int \frac{x-2}{(x-1)(x+1)}\,dx$$

$$= \frac{1}{2}x^2+x+\int \left[\frac{A}{x-1}+\frac{B}{x+1}\right]dx$$

$$\begin{array}{r|l} & x+1 \\ \hline x^2-1 & x^3+x^2-3 \\ & -(x^3-x) \\ \hline & x^2+x-3 \\ & -(x^2-1) \\ \hline & x-2 \end{array}$$

<u>Ex</u>: $\int \frac{x^2+4}{x^2+x}\,dx$

$$= \int \left[1+\frac{-x+4}{x^2+x}\right]dx$$

$$= \int \left[1+\frac{4-x}{x(x+1)}\right]dx$$

$$= x+\int \left[\frac{A}{x}+\frac{B}{x+1}\right]dx$$

$$\begin{array}{r|l} & 1 \\ \hline x^2+x & x^2+4 \\ & -(x^2+x) \\ \hline & -x+4 \end{array}$$