$$\boxed{\text{Why is } \int \tan x \, dx = \ln|\sec x| + c \text{ ?}}$$

$$\int \tan x \, dx = \int \frac{\sin x}{\cos x} \, dx$$

(Let $u = \cos x \xrightarrow{D} du = -\sin x \, dx$

$\rightarrow -du = \sin x \, dx$)

$$= -\int \frac{1}{u} \, du = -\ln|u| + c$$

$$= -\ln|\cos x| + c = \ln|\cos x|^{-1} + c$$

$$= \ln|\sec x| + c.$$

$$\boxed{\text{Why is } \int \sec x \, dx = \ln|\sec x + \tan x| + c \text{ ?}}$$

$$\int \sec x \, dx = \int \sec x \cdot \frac{\sec x + \tan x}{\sec x + \tan x} \, dx$$

$$= \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} \, dx$$

(Let $u = \sec x + \tan x \xrightarrow{D} du = (\sec x \tan x + \sec^2 x) \, dx$)

$$= \int \frac{1}{u} \, du = \ln|u| + c$$

$$= \ln|\sec x + \tan x| + c.$$