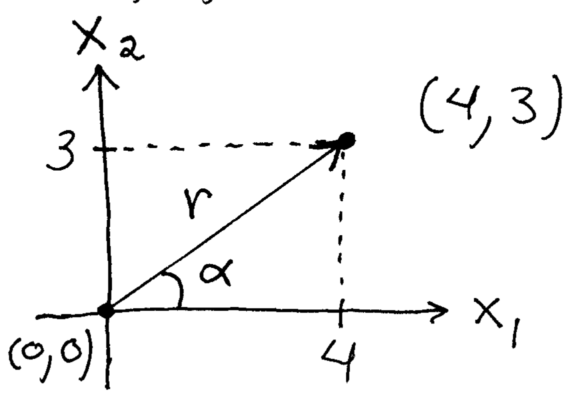
Vectors in $\mathbb{R}^2$

Example: Consider the point $(4,3)$ in the Cartesian Plane.



It's distance from the origin is

$$r = \sqrt{4^2+3^2} = 5,$$

and forms an angle α, where $\tan\alpha = \frac{3}{4}$, i.e, $\alpha = \arctan\left(\frac{3}{4}\right)$.

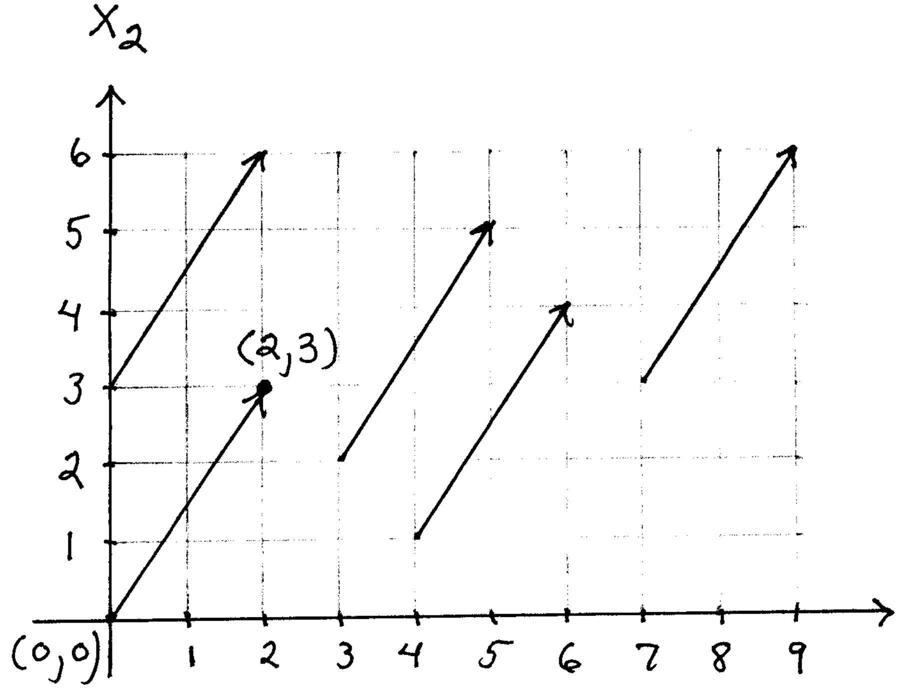
Definition: A 2×1 matrix is called a vector in $\mathbb{R}^2$.

Example: Vectors: $\begin{bmatrix} 4 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 1/2 \\ 3/4 \end{bmatrix}$

Note: 1.) A vector can also be represented by an arrow with length (magnitude) and direction (angle).

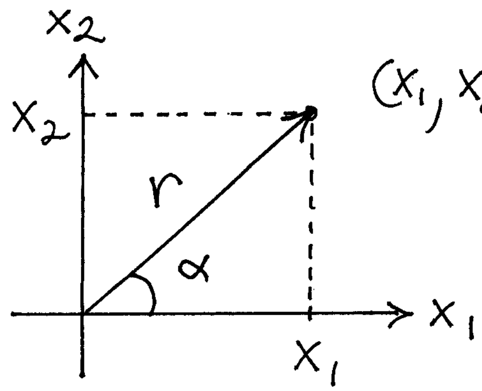
2.) Any two vectors with the same length and direction are considered equal.

Ex:



all 5 vectors are equal to the vector $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

Definition: Consider the vector determined by the point (x_1, x_2):



Cartesian form: $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

We also know that

$r = \sqrt{x_1^2 + x_2^2}$, $\alpha = \arctan\left(\frac{x_2}{x_1}\right)$;

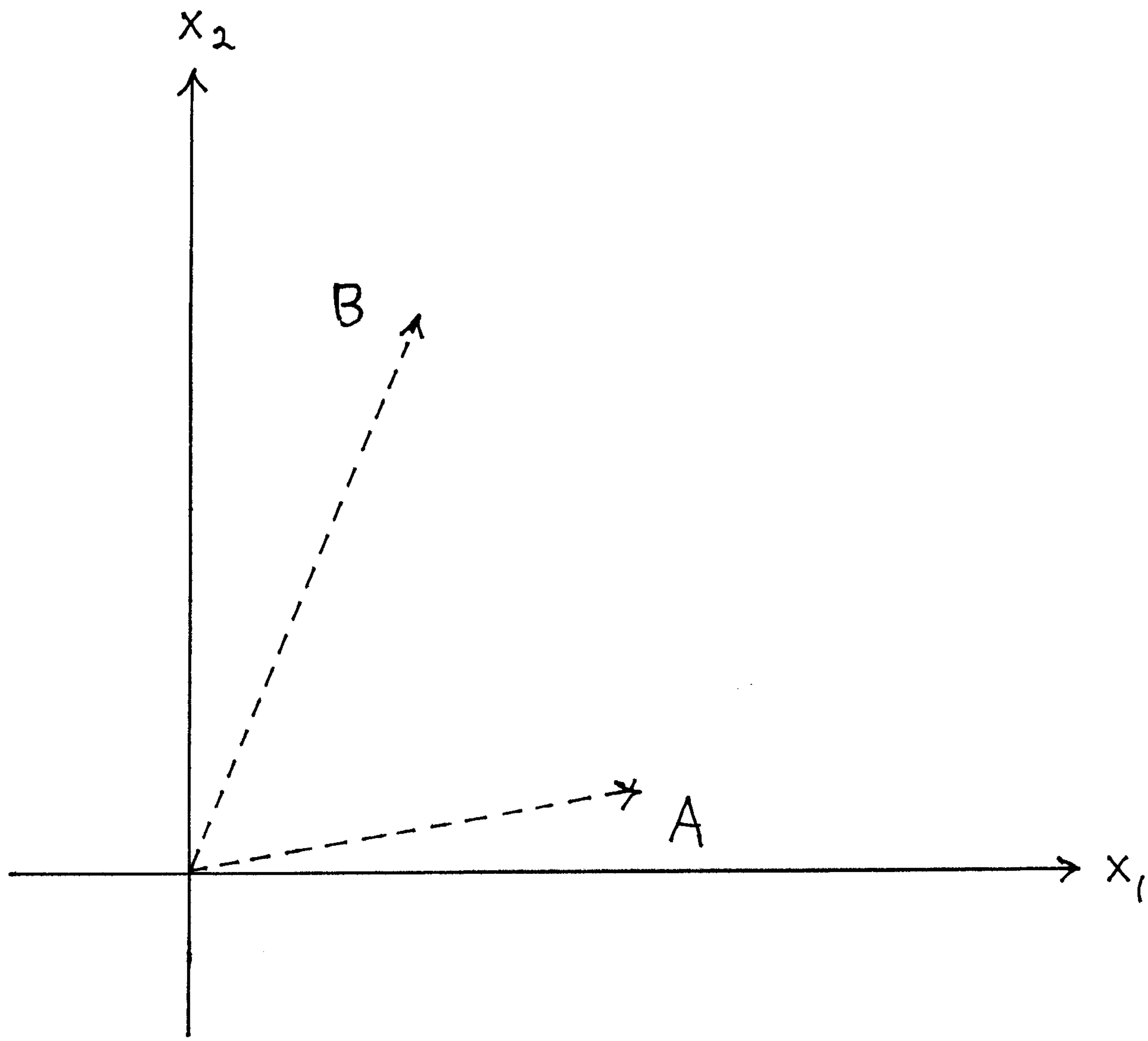
and $\cos\alpha = \frac{x_1}{r} \longrightarrow x_1 = r\cos\alpha$,

$\sin\alpha = \frac{x_2}{r} \longrightarrow x_2 = r\sin\alpha$; then a second form of a vector is

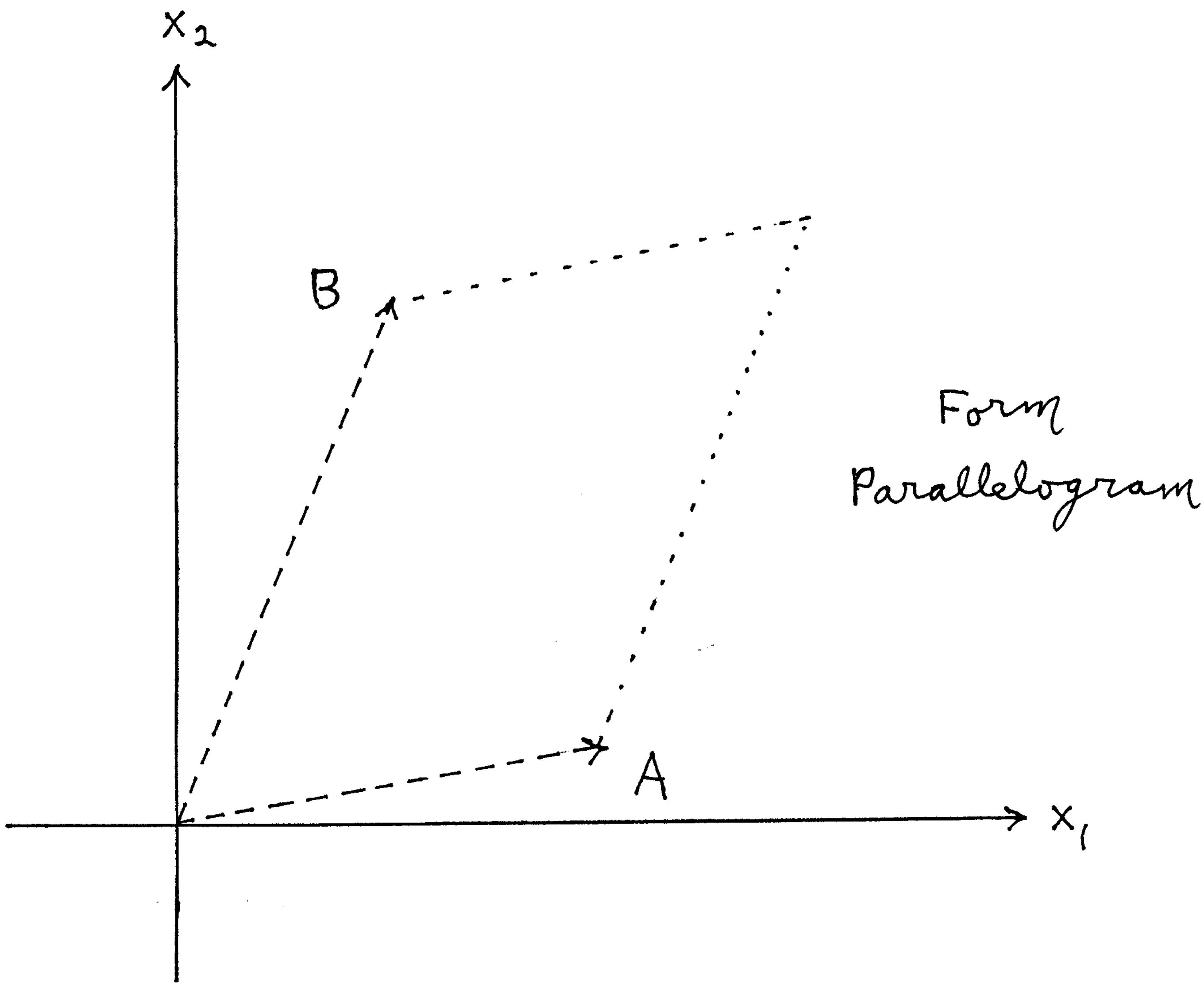
Polar form: $\begin{bmatrix} r\cos\alpha \\ r\sin\alpha \end{bmatrix}$, where

r is magnitude and α is direction.

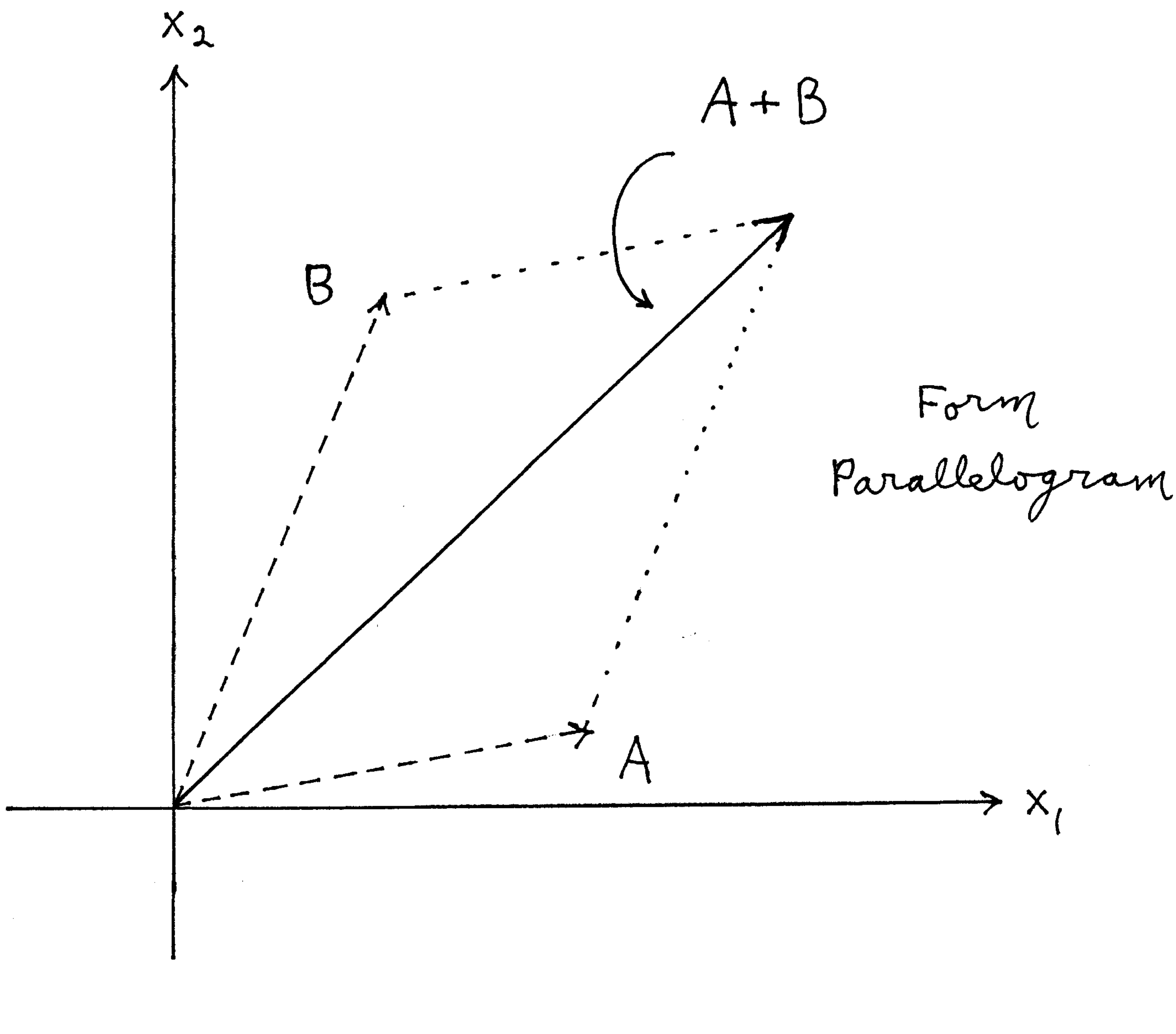
The Geometry of Vectors $A+B$ and $A-B$



The Geometry of Vectors $A+B$ and $A-B$



The Geometry of Vectors $A+B$ and $A-B$



The Geometry of Vectors $A+B$ and $A-B$