Math 17B
Kouba
Exam 1

Your Name : KEY

Your EXAM ID Number ____________

1. PLEASE DO NOT TURN THIS PAGE UNTIL TOLD TO DO SO.

2. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO, IN ANY WAY, ASSIST ANOTHER PERSON IN THE COMPLETION OF THIS EXAM. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO COPY ANSWERS FROM ANOTHER STUDENT'S EXAM. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO HAVE ANOTHER STUDENT TAKE YOUR EXAM FOR YOU. PLEASE KEEP YOUR OWN WORK COVERED UP AS MUCH AS POSSIBLE DURING THE EXAM SO THAT OTHERS WILL NOT BE TEMPTED OR DISTRACTED. THANK YOU FOR YOUR COOPERATION.

3. No notes, books, or classmates may be used as resources for this exam. YOU MAY USE A CALCULATOR ON THIS EXAM.

4. Read directions to each problem carefully. Show all work for full credit. In most cases, a correct answer with no supporting work will receive LITTLE or NO credit. What you write down and how you write it are the most important means of your getting a good score on this exam. Neatness and organization are also important.

5. Make sure that you have 7 pages, including the cover page.

6. You will be graded on proper use of integral and derivative notation.

7. You will be graded on proper use of limit notation.

8. You have until 10:50 a.m. to finish the exam.

1.) (7 pts. each) Use any method to integrate each of the following. DO NOT SIMPLIFY answers.

a.) $\int \cos 3x + \sin x \; dx$

$= \frac{1}{3} \sin 3x - \cos x + c$

b.) $\int \frac{x^3}{2+x^4} \, dx$ $\quad$ (Let $u = 2 + x^4 \rightarrow du = 4x^3 dx$ $\rightarrow \frac{1}{4} du = x^3 dx$)

$= \frac{1}{4} \int \frac{1}{u} du = \frac{1}{4} \ln|u| + c = \frac{1}{4} \ln|2+x^4| + c$

c.) $\int (x-2)(x+1)^{10} \, dx$ $\quad$ (Let $u = x+1$, $x = u-1$, $du = dx$)

$= \int ((u-1)-2) u^{10} du = \int (u-3) u^{10} du = \int (u^{11} - 3u^{10}) du$

$= \frac{1}{12} u^{12} - 3 \cdot \frac{1}{11} u^{11} + c = \frac{1}{12}(x+1)^{12} - \frac{3}{11}(x+1)^{11} + c$

d.) $\int \tan^2 x \, \sec^2 x \, dx$ $\quad$ (Let $u = \tan x \rightarrow du = \sec^2 x \, dx$)

$= \int u^2 du = \frac{1}{3} u^3 + c = \frac{1}{3} (\tan x)^3 + c$

e.) $\int \frac{x-1}{x^2+2x} \, dx = \int \frac{x-1}{x(x+2)} dx = \int \left[\frac{A}{x} + \frac{B}{x+2} \right] dx$

$\{ x - 1 = A(x+2) + Bx$
Let $\underline{x=0}$: $-1 = 2A \rightarrow A = -\frac{1}{2}$
Let $\underline{x=-2}$: $-3 = -2B$
$\rightarrow B = \frac{3}{2} \}$

$= \int \left[\frac{-\frac{1}{2}}{x} + \frac{\frac{3}{2}}{x+2} \right] dx$

$= -\frac{1}{2} \ln|x| + \frac{3}{2} \ln|x+2| + c$

f.) $\int \frac{\sqrt{x}}{4+x} \, dx$ $\quad$ (Let $x = u^2 \rightarrow dx = 2u \, du$)

$= \int \frac{\sqrt{u^2} \cdot 2u}{4+u^2} du = \int \frac{2u^2}{u^2+4} du$

$$\begin{array}{r} 2 \\ u^2+4 \overline{\smash{\big)}\, 2u^2 } \\ -(2u^2+8) \\ \hline -8 \end{array}$$

$= \int \left[2 - \frac{8}{u^2+2^2} \right] du = 2u - 8 \cdot \frac{1}{2} \arctan \frac{u}{2} + c$

$= 2\sqrt{x} - 4 \arctan\left(\frac{\sqrt{x}}{2}\right) + c$

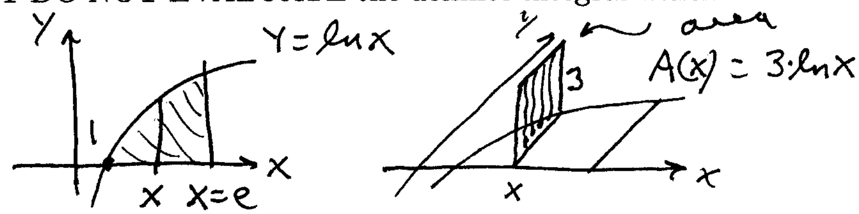
2.) (6 pts.) If $y = f(x)$ is an odd function and $\int_3^4 f(x)\,dx = 5$, then what is the value of

$\int_{-3}^4 f(x)\,dx$?

$$\int_{-3}^{4} f(x)\,dx = \underbrace{\int_{-3}^{3} f(x)\,dx}_{0 \text{ since } f \text{ odd}} + \int_{3}^{4} f(x)\,dx$$

$$= 0 + 5 = 5$$

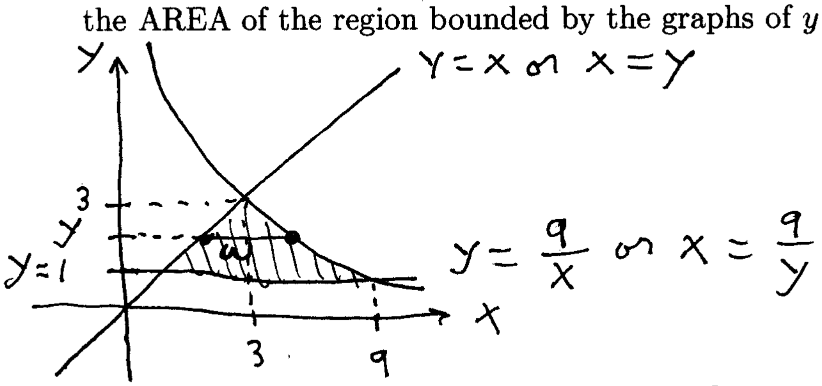
3.) (8 pts.) The base of a three-dimensional solid lies in the region bounded by the graphs of $y = 0$, $y = \ln x$, and $x = e$. Cross-sections of the solid at x perpendicular to the x-axis are rectangles of height 3. SET UP BUT DO NOT EVALUATE the definite integral which represents the VOLUME of the solid.



$$\text{Volume} = \int_1^e A(x)\,dx$$

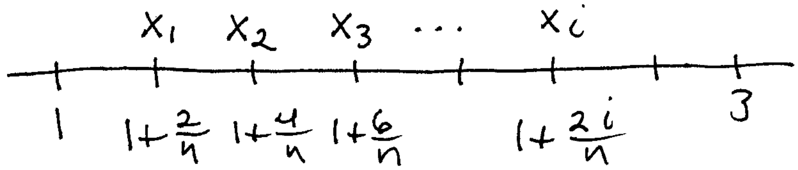
$$= \int_1^e 3 \ln x\,dx$$

4.) (8 pts.) SET UP BUT DO NOT EVALUATE the definite integral(s) which represents the AREA of the region bounded by the graphs of $y = x$, $y = \dfrac{9}{x}$, and $y = 1$.



$$\text{Area} = \int_1^3 w\,dy = \int_1^3 \left[\frac{9}{y} - y\right] dy$$

5.) (8 pts.) Use the limit definition of the definite integral (for convenience, you may choose equal subdivisions and right-hand endpoints) to evaluate $\int_1^3 x^2\,dx$.



$$\Delta x_i = \frac{3-1}{n} = \frac{2}{n}\,,$$

$$x_i = 1 + \frac{2i}{n}$$

$$\int_1^3 x^2\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\cdot \Delta x_i$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(1+\frac{2i}{n}\right)^2 \cdot \frac{2}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(1+\frac{4i}{n}+\frac{4i^2}{n^2}\right)\cdot \frac{2}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{2}{n}+\frac{8}{n^2}i+\frac{8}{n^3}i^2\right)$$

$$= \lim_{n\to\infty} \left[\frac{2}{n}\cdot\left(\sum_{i=1}^{n} 1\right)+\frac{8}{n^2}\cdot\left(\sum_{i=1}^{n} i\right)+\frac{8}{n^3}\left(\sum_{i=1}^{n} i^2\right)\right]$$

$$= \lim_{n\to\infty} \left[\frac{2}{\cancel{n}}\cdot(\cancel{n}) + \frac{\overset{4}{\cancel{8}}}{n^2}\cdot\frac{n(n+1)}{\cancel{2}} + \frac{\overset{4}{\cancel{8}}}{n^3}\cdot\frac{n(n+1)(2n+1)}{\underset{3}{\cancel{6}}}\right]$$

$$= \lim_{n\to\infty} \left[2 + 4\cdot\frac{\cancel{n}}{\cancel{n}}\cdot\frac{n+1}{n} + \frac{4}{3}\cdot\frac{\cancel{n}}{\cancel{n}}\cdot\frac{n+1}{n}\cdot\frac{2n+1}{n}\right]$$

$$= \lim_{n\to\infty} \left[2 + 4\cdot\left(1+\cancelto{0}{\frac{1}{n}}\right) + \frac{4}{3}\left(1+\cancelto{0}{\frac{1}{n}}\right)\left(2+\cancelto{0}{\frac{1}{n}}\right)\right]$$

$$= 2 + 4(1) + \frac{4}{3}\cdot(1)(2)$$

$$= \frac{18}{3} + \frac{8}{3} = \frac{26}{3}$$

6.) (7 pts.) The temperature T of a room at time t minutes is $T(t) = \sqrt{16+t}$ ^oF . Find the *average* temperature of the room from $t = 0$ to $t = 20$ minutes.

$$AVE = \frac{1}{20-0}\int_0^{20}\sqrt{16+t}\,dt = \frac{1}{20}\cdot\frac{2}{3}(16+t)^{3/2}\Big|_0^{20}$$

$$= \frac{1}{30}(36)^{3/2} - \frac{1}{30}(16)^{3/2}$$

$$= \frac{216}{30} - \frac{64}{30} = \frac{152}{30} = \frac{76}{15} \approx 5.07\ ^oF$$

7.) (8 pts.) Find the length of the graph of $y = (3/2)x^{2/3}$ on the interval $[1, 8]$.

$$y' = \cancel{\tfrac{3}{2}}\cdot\cancel{\tfrac{2}{3}}x^{-1/3} = \frac{1}{x^{1/3}}$$ ~~then~~

$$ARC = \int_1^8\sqrt{1+(y')^2}\,dx = \int_1^8\sqrt{1+\left(\frac{1}{x^{1/3}}\right)^2}\,dx$$

$$= \int_1^8\sqrt{1+\frac{1}{x^{2/3}}}\,dx = \int_1^8\sqrt{\frac{x^{2/3}+1}{x^{2/3}}}\,dx = \int_1^8\frac{\sqrt{x^{2/3}+1}}{x^{1/3}}\,dx$$

(~~let~~ $u = x^{2/3}+1 \rightarrow du = \frac{2}{3}x^{-1/3}dx \rightarrow \frac{3}{2}du = \frac{1}{x^{1/3}}dx$

and $x: 1 \rightarrow 8$ ~~so~~ $u: 2 \rightarrow 5$)

$$= \frac{3}{2}\int_2^5\sqrt{u}\,du = \cancel{\tfrac{3}{2}}\cdot\cancel{\tfrac{2}{3}}u^{3/2}\Big|_2^5$$

$$= 5^{3/2} - 2^{3/2}$$

8.) (6 pts.) Let $F(x) = x \cdot \int_1^{x^2} e^{\sqrt{t}}\, dt$. Find $F'(1)$.

$$F'(x) = x \cdot D\left[\int_1^{x^2} e^{\sqrt{t}}\, dt\right] + (1) \cdot \int_1^{x^2} e^{\sqrt{t}}\, dt$$

$$= x \cdot e^{\sqrt{x^2}} \cdot 2x + \int_1^{x^2} e^{\sqrt{t}}\, dt$$

$$= 2x^2 e^x + \int_1^{x^2} e^{\sqrt{t}}\, dt \text{ , then}$$

$$F'(1) = 2(1)^2 e^1 + \underbrace{\int_1^1 e^{\sqrt{t}}\, dt}_{0}$$

$$= 2e$$

9.) (7 pts.) Use integration by parts twice with a twist to determine $\int \sin 3x \ \cos x \ dx$.

(Let $u = \sin 3x$, $dv = \cos x\, dx$

$\to du = 3\cos 3x\, dx$, $v = \sin x$)

$$\int \sin 3x \cdot \cos x\, dx = \sin 3x \cdot \sin x - 3\int \cos 3x \cdot \sin x\, dx$$

(Let $u = \cos 3x$, $dv = \sin x\, dx$

$\to du = -3\sin 3x$, $v = -\cos x$)

$$= \sin 3x \cdot \sin x - 3\left[-\cos 3x \cdot \cos x - 3\int \sin 3x \cos x\, dx\right]$$

$$= \sin 3x \cdot \sin x + 3\cos 3x \cdot \cos x + 9\int \sin 3x \cos x\, dx$$

(now twist) $\to$

$$-8\int \sin 3x \cdot \cos x\, dx = \sin 3x \cdot \sin x + 3\cos 3x \cdot \cos x + C$$

$$\to \int \sin 3x \cdot \cos x\, dx = \frac{-1}{8}\sin 3x \cdot \sin x - \frac{3}{8}\cos 3x \cos x + C$$

The following EXTRA CREDIT problem is OPTIONAL. It is worth 10 points.

1.) Use any method to integrate : $\int \sin(\ln x)\,dx$

(Let $u = \ln x \rightarrow x = e^u \rightarrow dx = e^u\,du$)

$\int \sin(\ln x)\,dx = \int e^u \sin u\,du$ (Now do integration by parts twice with a twist)

$\{$Let $w = e^u$, $dV = \sin u\,du$
$\rightarrow dw = e^u\,du$, $V = -\cos u\}$

$= -e^u \cos u - -\int e^u \cos u\,du$

$= -e^u \cos u + \int e^u \cos u\,du$ $\{$Let $w = e^u$, $dV = \cos u\,du$
$\rightarrow dw = e^u\,du$, $V = \sin u\}$

$= -e^u \cos u + \left[e^u \sin u - \int e^u \sin u\,du\right]$

(now twist) $\rightarrow$

$2\int e^u \sin u\,du = -e^u \cos u + e^u \sin u + C \rightarrow$

$\int e^u \sin u\,du = \frac{-1}{2} e^u \cos u + \frac{1}{2} e^u \sin u + C \rightarrow$

$\int \sin(\ln x)\,dx = \int e^u \sin u\,du$

$= \frac{-1}{2} e^{\ln x} \cdot \cos(\ln x) + \frac{1}{2} e^{\ln x} \cdot \sin(\ln x) + C$

$= \frac{-1}{2} x \cdot \cos(\ln x) + \frac{1}{2} x \cdot \sin(\ln x) + C$