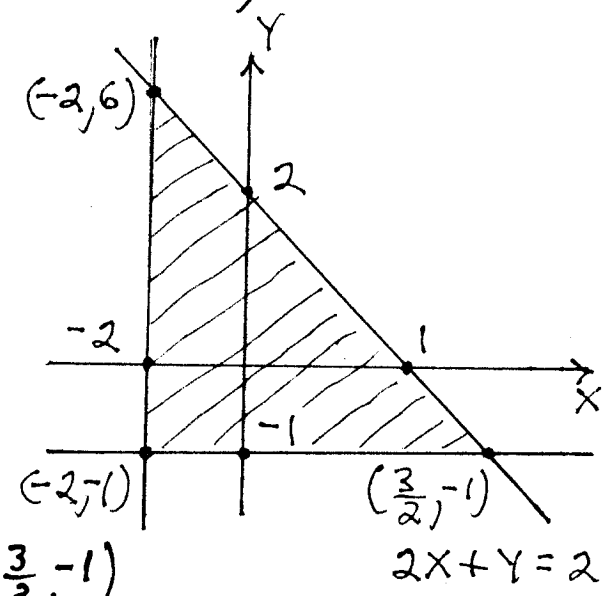
Math 17C
Kouba

Extrema for $z=f(x,y)$ on a Bounded Domain

Example: Find the absolute extrema on the surface $z=x^2+y^2$ defined on region R bounded by the graphs of $y=-1$, $x=-2$, and $2x+y=2$.



Find critical pts.:

$z=x^2+y^2 \rightarrow z_x=2x=0$
$\rightarrow x=0$ and $z_y=2y=0$
$\rightarrow y=0$ so crit. pt. is $(0,0)$;

Corners: $(-2,6)$, $(-2,-1)$, $(\frac{3}{2},-1)$

along $x=-2$: $z=x^2+y^2=(-2)^2+y^2=4+y2$
$\rightarrow z'=2y=0 \rightarrow y=0$ so crit. pt. is $(-2,0)$;

along $y=-1$: $z=x^2+y^2=x^2+(-1)^2=x^2+1 \rightarrow$
$z'=2x=0 \rightarrow x=0$ so crit. pt. is $(0,-1)$;

along $2x+y=2 \rightarrow y=2-2x$:

$z=x^2+y^2=x^2+(2-2x)^2 \rightarrow$
$z'=2x+2(2-2x)(-2)=2x-8+8x=10x-8=0$

$\rightarrow x = \frac{4}{5}$ so crit. pt. is $\left(\frac{4}{5}, \frac{2}{5}\right)$;

(x,y)	z-value	
$(0,0)$	0	← ABS MIN
$(-2,6)$	40	← ABS MAX
$(-2,-1)$	5	
$(3/2,-1)$	13/4	
$(-2,0)$	4	
$(0,-1)$	1	
$\left(\frac{4}{5}, \frac{2}{5}\right)$	4/5	