

Section 12.2

- 1.) $\{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$
- 2.) $\{11, 12, 13, 14, 15, 16, 21, 22, 23, 24, 25, 26, 31, 32, 33, 34, 35, 36, 41, 42, 43, 44, 45, 46, 51, 52, 53, 54, 55, 56, 61, 62, 63, 64, 65, 66\}$
- 3.) $\{\{1,2\}, \{1,3\}, \{1,4\}, \{1,5\}, \{2,3\}, \{2,4\}, \{2,5\}, \{3,4\}, \{3,5\}, \{4,5\}\}$
- 4.) $\{\{1,2,3,4,5\}, \{1,2,3,4,6\}, \{1,2,3,5,6\}, \{1,2,4,5,6\}, \{1,3,4,5,6\}, \{2,3,4,5,6\}\}$
- 5.) a.) $A \cup B = \{1, 2, 3, 5\}$
b.) $A \cap B = \{1, 3\}$
- 6.) a.) $A^c = \{2, 4, 6\}$
b.) $(A^c)^c = \{2, 4, 6\}^c = \{1, 3, 5\} = A$
- 7.) $(A \cup B)^c = \{1, 2, 3, 5\}^c = \{4, 6\}$
- 8.) $A \cap B = \{1, 3\} \neq \emptyset$, so A and B are not disjoint
- 9.) $P(1) + P(2) + P(3) + P(4) + P(5) = 1 \rightarrow$

$$\begin{aligned}
 P(5) &= 1 - P(1) - P(2) - P(3) - P(4) \\
 &= 1 - 0.1 - 0.2 - 0.05 - 0.05 \\
 &= 1 - 0.4 = 0.6
 \end{aligned}$$

$$\begin{aligned}
 10.) \text{ a.) } P(A) &= P(1) + P(3) + P(5) \\
 &= 0.1 + 0.05 + 0.6 = 0.75
 \end{aligned}$$

$$\begin{aligned}
 \text{b.) } P(B) &= P(2) + P(3) + P(4) \\
 &= 0.2 + 0.05 + 0.05 = 0.3
 \end{aligned}$$

$$11.) P(A^c) = 1 - P(A) = 1 - 0.75 = 0.25$$

$$\begin{aligned}
 12.) P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\
 &= 0.75 + 0.3 - P\{3\} \\
 &= 0.75 + 0.3 - 0.05 \\
 &= 1
 \end{aligned}$$

$$\begin{aligned}
 17.) \quad & \underline{P(A \cup B) = P(A) + P(B) - P(A \cap B)} \quad \text{and} \\
 & P(A^c \cap B^c) = P((A \cup B)^c) = 1 - P(A \cup B) \rightarrow \\
 & \underline{P(A \cup B) = 1 - P(A^c \cap B^c)} \quad ; \quad \text{then (SUB)} \\
 & 1 - P(A^c \cap B^c) = P(A) + P(B) - P(A \cap B) \rightarrow \\
 & 1 - 0.2 = 0.4 + P(B) - 0.1 \rightarrow \\
 & P(B) = 1 - 0.2 - 0.3 = 0.5
 \end{aligned}$$

$$\begin{aligned}
 18.) \quad & \text{See formulas in 13.). Then} \\
 & P(A \cap B) = P(A) + P(B) - P(A \cup B) \rightarrow \\
 & P(A \cap B) = 0.4 + 0.4 - 0.7 = 0.1 ;
 \end{aligned}$$

$$P(A^c \cap B^c) = 1 - P(A \cup B) = 1 - 0.7 = 0.3$$

21.) Sample Space: $\{HH, HT, TH, TT\} \rightarrow$
 $P(\text{at least 1 H}) = \frac{3}{4}$

22.) Sample Space:
 $\{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\} \rightarrow$
 $P(\text{no H's}) = \frac{1}{8}$

25.) $\frac{6 \cdot 6}{1st \ 2nd} = 36$ outcomes possible

Let A: 4 on 1st roll

B: 4 on 2nd roll then

$$\begin{aligned} P(\text{at least one 4}) &= P(A \cup B) \\ &= P(A) + P(B) - P(A \cap B) \\ &= \frac{1}{6} + \frac{1}{6} - \frac{1}{36} = \frac{12}{36} - \frac{1}{36} = \frac{11}{36} \end{aligned}$$

26.) $\frac{6 \cdot 6}{1st \ 2nd} = 36$ outcomes possible;

both even: $\frac{3 \cdot 3}{1st \ 2nd} = 9$ ways,

both odd: $\frac{3 \cdot 3}{1st \ 2nd} = 9$ ways, so

$$\begin{aligned} P(\text{sum is even: both even or both odd}) \\ &= \frac{9+9}{36} = \frac{18}{36} = \frac{1}{2} \end{aligned}$$

27.) Sample Space : 36 outcomes, and
 let A : event that 1st # > 2nd # ; then
 $A = \{65, 64, 63, 62, 61, 54, 53, 52, 51, 43, 42,$
 $41, 32, 31, 21\} \rightarrow$
 $P(A) = \frac{15}{36} = \frac{5}{12}$

28.) Sample Space : 36 outcomes, and
 let A : event that each # is > 4 ; then
 $A = \{55, 56, 65, 66\} \rightarrow$
 $P(A) = \frac{4}{36} = \frac{1}{9}$

29.) (See p. 801) $P(\text{white}) = P(\text{cc}) = \frac{1}{4}$

30.) (See p. 801)

a.)

	c	c
c	Cc	cc
c	Cc	cc

b.) $P(\text{red}) = P(\text{Cc}) = \frac{2}{4} = \frac{1}{2}$

31.)

	A	a
A	AA	Aa
a	Aa	aa

$P(Aa) = \frac{2}{4} = \frac{1}{2}$

32.)

	A	A
A	AA	AA
a	Aa	Aa

$P(AA) = \frac{2}{4} = \frac{1}{2}$

33.) Sample Space :

$\{GGG, GGB, GBG, BGG, GBB, BGB, BBG, BBB\}$

$$P(3 \text{ girls}) = \frac{1}{8}$$

XX: normal female

X(b1)X: normal female

XY: normal male

X(b1)Y: color blind male

36.) Female X(b1) X

Male X	X(b1)X	XX
Male Y	X(b1)Y	XY

$$P(\text{colorblind male}) = \frac{1}{4}$$

40.) Sample Space : 5B, 3G balls ;
pick 3 at random, then

$$P(\text{at least 2G}) = P(2G) + P(3G)$$

$$= \frac{C(3,2) \cdot C(5,1)}{C(8,3)} + \frac{C(3,3)}{C(8,3)} = \frac{3 \cdot 5}{56} + \frac{1}{56} = \frac{16}{56} = \frac{2}{7}$$

Select 2 cards:

$$41.) P(2 \text{ spades}) = \frac{C(13,2)}{C(52,2)} = \frac{78}{1326} = \frac{1}{17}$$

42.) Select 5 cards:

$$P(4 \text{ aces}) = \frac{C(4,4) \cdot C(48,1)}{C(52,5)}$$
$$= \frac{1 \cdot 48}{2,598,960} = \frac{1}{54,145}$$

43.) Sample Space: $\{4G, 6B, 2R\}$,
pick 3 at random, then

$$P(1G, 1B, 1R) = \frac{C(4,1) \cdot C(6,1) \cdot C(2,1)}{C(12,3)}$$
$$= \frac{4 \cdot 6 \cdot 2}{220} = \frac{12}{55}$$

44.) Sample Space: $\{3G, 5B, 4R\}$,
pick 3 at random, then:

$$P(3G) = \frac{C(3,3)}{C(12,3)} = \frac{1}{220}$$

$$P(3B) = \frac{C(5,3)}{C(12,3)} = \frac{10}{220} = \frac{1}{22}$$

$$P(3R) = \frac{C(4,3)}{C(12,3)} = \frac{4}{220} = \frac{1}{55}$$

$$P(3 \text{ same color}) = \frac{4}{455} + \frac{20}{455} + \frac{10}{455}$$

$$= \frac{34}{455}$$

45.) Select 4 cards :

$$P(\text{at least 1 A}) = 1 - P(\text{no A})$$

$$= 1 - \frac{C(48, 4)}{C(52, 4)} = 1 - \frac{194,580}{270,725} \approx 0.281$$

46.) Select 4 cards :

$$\frac{C(13, 1)}{\substack{\text{pick} \\ \text{face} \\ \text{value}}} \cdot \frac{C(4, 2)}{\substack{\text{pick} \\ \text{the} \\ \text{pair}}} \cdot \frac{C(12, 2)}{\substack{\text{pick} \\ \text{2 face} \\ \text{values}}} \cdot \frac{C(4, 1)}{\substack{\text{pick} \\ \text{card} \\ 3}} \cdot \frac{C(4, 1)}{\substack{\text{pick} \\ \text{card} \\ 4}} = (13)(6)(66)(4)(4)$$

$$= 82,368 \rightarrow$$

$$P(\text{exactly 1 pair}) = \frac{82,368}{C(52, 4)} \approx 0.304$$

47.) (52 cards: 13 red hearts, 13 red diamonds, 13 black spades, 13 black clubs)

Select 13 cards :

$$P(13 \text{ red}) = \frac{C(26, 13)}{C(52, 13)} = \frac{10,400,600}{635,013,559,600} \approx 0.000016$$

48.) Select 4 cards :

$$P(1\heartsuit, 1\spadesuit, 1\clubsuit, 1\spadesuit)$$

$$= \frac{C(13,1) \cdot C(13,1) \cdot C(13,1) \cdot C(13,1)}{C(52,4)}$$

$$C(52,4)$$

$$= \frac{13 \cdot 13 \cdot 13 \cdot 13}{270,725} \approx 0.105$$

49.) Select 5 cards : pick exactly 2 pairs \rightarrow

$$\frac{C(13,2) \cdot C(4,2) \cdot C(4,2) \cdot C(44,1)}{\substack{\text{pick 2} \\ \text{face} \\ \text{values} \quad \text{pick} \\ \text{pair 1} \quad \text{pick} \\ \text{pair 2} \quad \text{pick} \\ \text{5th} \\ \text{card}}} = 78 \cdot 6 \cdot 6 \cdot 44 = 123,552 \rightarrow$$

$$P(\text{exactly 2 pairs}) = \frac{123,552}{C(52,5)} = \frac{123,552}{2,598,960} \approx 0.048$$

50.) Select 5 cards :

$$\frac{C(13,1) \cdot C(12,1) \cdot C(4,3) \cdot C(4,2)}{\substack{\text{pick} \\ \text{triple} \\ \text{face} \\ \text{value} \quad \text{pick} \\ \text{pair} \\ \text{face} \\ \text{value} \quad \text{pick} \\ \text{triple} \quad \text{pick} \\ \text{pair}}} = 13 \cdot 12 \cdot 4 \cdot 6 = 3744 \rightarrow$$

$$P(\text{full house}) = \frac{3744}{C(52,5)} = \frac{3744}{2,598,960} \approx 0.0014$$

51.) N fish, 100 marked :

a.) 3 marked fish out of 10 :

$$\frac{C(100, 3)}{\substack{\text{pick 3} \\ \text{marked} \\ \text{fish}}} \cdot \frac{C(N-100, 7)}{\substack{\text{pick 7} \\ \text{unmarked} \\ \text{fish}}}$$

Sample Space size: $C(N, 10) \rightarrow$

$$P(3 \text{ marked fish out of } 10) = \frac{C(100, 3) \cdot C(N-100, 7)}{C(N, 10)}$$