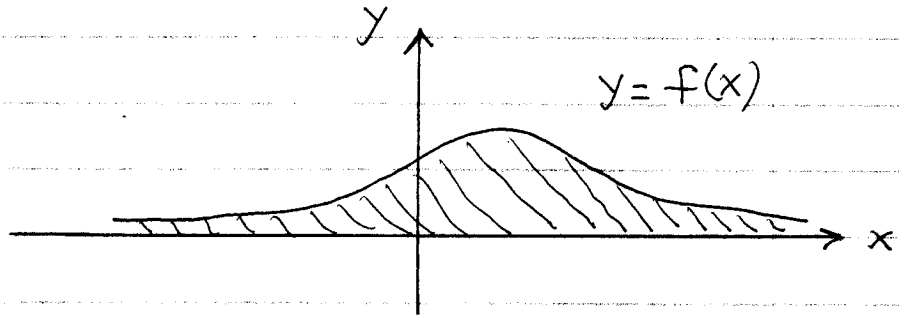
Math 17C
Kouba

Probability Density Function

Def: Let X be a continuous random variable with values x, $-\infty < x < \infty$. A *probability density function* f is a function with values $f(x)$ and with properties:

1.) $f(x) \geq 0$ for all values of x

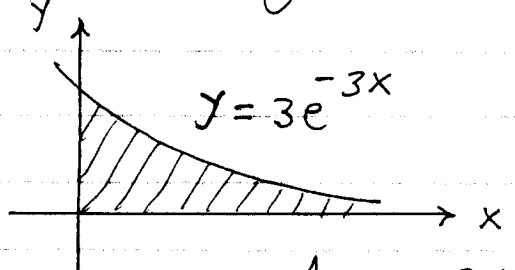
2.) $\int_{-\infty}^{\infty} f(x)\,dx = 1$



Ex: Show that

$$f(x) = \begin{cases} 3e^{-3x}, & \text{if } x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

is a probability density function:



Clearly $f(x) \geq 0$ and

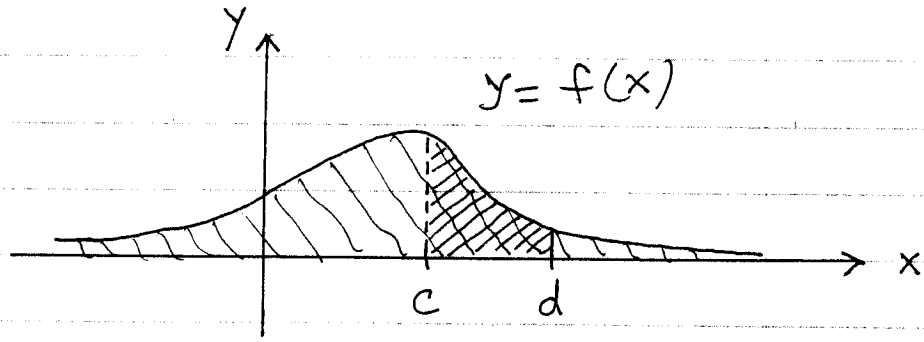
$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{0}^{\infty} 3e^{-3x}\,dx$$

$$= \lim_{A \to \infty} \int_0^A 3e^{-3x}\,dx = \lim_{A \to \infty} \frac{3e^{-3x}}{-3}\Big|_0^A$$

$$= \lim_{A\to\infty} \left(-e^{-3A} - -e^{0}\right) = \lim_{A\to\infty} \left(\frac{-1}{e^{3A}} + 1\right)$$

$$= 0 + 1 = 1.$$

FACTS: Let f be a probability density function for a continuous random variable X. Then



1.) $P(c \le x \le d) = \int_c^d f(x)\,dx$

2.) the expected value (mean) of X is $\mu = E(X) = \int_{-\infty}^{\infty} x\, f(x)\,dx$

3.) the variance of X is

$$\text{var}(X) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$$

OR $\text{var}(X) = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2$

4.) the standard deviation of $X = \sqrt{\text{var}(X)}$