Math 21A
Kouba
Continuity

**Def**: Function $Y=f(x)$ is continuous at $x=a$ if

i.) $f(a)$ exists,

ii) $\lim_{x\to a} f(x)$ exists (finite),

and iii) $\lim_{x\to a} f(x) = f(a)$.

**Fact**: Sums, differences, products, quotients (denominator $\neq 0$), and compositions of continuous functions are continuous.

**Fact**: Every polynomial is continuous for all x-values.

**Ex**: Let $f(x) = 7x^5 - x^4 + 2x^3 - x + 20$; $f$ is continuous for all values of $x$ since it is a polynomial.

**Ex**: Let $f(x) = \dfrac{x^2-5x+6}{2x^2+x-3}$; since $Y = x^2 - 5x + 6$ (parabola) and $Y = 2x^2 + x - 3$ (parabola) are continuous for all x-values, it follows that $f(x) = \dfrac{x^2-5x+6}{2x^2+x-3}$ is continuous for all x-values except where $Y = 2x^2 + x - 3 = (2x+3)(x-1) = 0$, that is, except at $x=1$ and $x = -3/2$.

**Ex**: Let $f(x) = (3+\sin x)^{50}$; since $g(x) = 3 + \sin x$ ($\sim\sim$) and $h(x) = x^{50}$ (polynomial) are continuous for all values of $x$, it follows that their composition $f(x) = h(g(x)) = (3+\sin x)^{50}$ is continuous for all values of $x$.