Math 21A
Kouba
The Plausibility of L'Hopital's Rule, The $\dfrac{0}{0}$ Case

<u>L'Hopital's Rule</u> ( $\dfrac{0}{0}$ Case) : If $\lim_{x \to a} f(x) = 0$, $\lim_{x \to a} g(x) = 0$, and $\lim_{x \to a} \dfrac{f'(x)}{g'(x)} = L$ (a finite number or $\pm\infty$), then $\lim_{x \to a} \dfrac{f(x)}{g(x)} = L$ .

Assume that $f, g, f'$, and $g'$ are continuous for all x-values in an interval containing $a$, so that

$$\lim_{x \to a} f(x) = f(a) = 0,$$

$$\lim_{x \to a} g(x) = g(a) = 0 \ ,$$

$$\lim_{x \to a} f'(x) = f'(a)$$

$$\text{and } \lim_{x \to a} g'(x) = g'(a) \ .$$

Then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f(x) - f(a)}{g(x) - g(a)}$$

$$= \lim_{x \to a} \frac{\frac{f(x)-f(a)}{x-a}}{\frac{g(x)-g(a)}{x-a}}$$

$$= \frac{f'(a)}{g'(a)}$$

$$= \lim_{x \to a} \frac{f'(x)}{g'(x)} = \text{L} \ .$$