Math 21A
Kouba
Discussion Sheet 3

1.) Evaluate the following limits.

a.) $\lim_{x \to 4} \dfrac{\sqrt{5+x}-3}{\sqrt{x}-2}$ b.) $\lim_{x \to \infty} (x - x\cos(4/x))$

c.) $\lim_{x \to 0} \dfrac{2x}{\sin x - x}$ (HINT : $\dfrac{\sin x}{x} \leq 1$.)

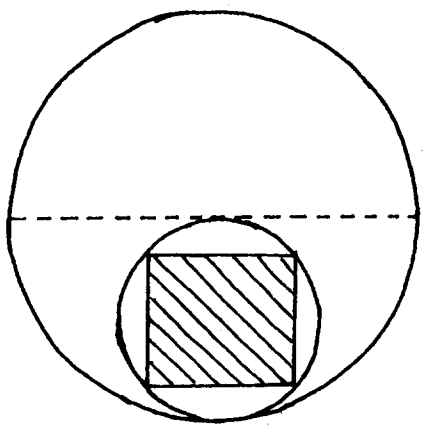
2.) Use one-sided limits and limits to infinity to find all vertical and horizontal asymptotes for the following functions. Use the asymptotes together with x- and y-intercepts to sketch graphs of each function.

a.) $y = \dfrac{x-2}{x^2-9}$ b.) $y = \dfrac{x+2}{x^2(x+1)}$

3.) Give an ε, δ -proof for $\lim_{x \to 1} \dfrac{x+3}{1+\sqrt{x}} = 2$.

4.) Use the IMVT to determine if the following equation is solvable. This is a writing exercise : $x^3 + x - \sqrt{x+4} = 0$.

5.) In the given diagram the smaller circle is the largest one that can be inscribed in the given semi-circle. If the larger circle has circumference 4π in., what is the area of the inscribed shaded square ?



6.) Determine the x-values for which the following function is continuous. It is not necessary to graph the function :

$$f(x) = \begin{cases} \dfrac{\sin 3x}{x}, & \text{if } x < 0 \\ 3.01, & \text{if } x = 0 \\ \dfrac{x-1}{\sqrt{x}-1}, & \text{if } 0 < x < 1 \\ 2, & \text{if } x \geq 1 \ . \end{cases}$$

7.) Use the limit definition of derivative,
$f'(x) = \lim_{h \to 0} \dfrac{f(x+h)-f(x)}{h}$, to differentiate each of the following functions.

a.) $f(x) = \cos x$

b.) $f(x) = \dfrac{7+x}{3x-5}$

c.) $f(x) = \sqrt{x^2+x}$

8.) Let $f(x) = \begin{cases} \sin 2x, & \text{if } x \geq 0 \\ 2x, & \text{if } x < 0 \end{cases}$. Use the limit definition of derivative to determine $f'(0)$

++

The following problem is for recreational purposes only.

9.) Without lifting your pencil, join all sixteen dots with six straight lines.