Math 21A Kouba An Extreme Example

The following function, which requires some knowledge of infinite series, represents a function which is *continuous* for all values of x, but *differentiable at no values of x*. The source is Principles of Mathematical Analysis, by Walter Rudin, McGraw-Hill, 1964, page 141.

7.18. Theorem. There exists a real continuous function on the real line which is nowhere differentiable.

**Proof:** Define

(44) 
$$\phi(x) = \begin{cases} x & (0 \le x \le 1), \\ 2 - x & (1 \le x \le 2) \end{cases}$$

and extend the definition of  $\phi(x)$  to all real x by requiring that

$$\phi(x+2)=\phi(x).$$

Then  $\phi$  is continuous on  $R^1$ . Define

(45) 
$$f(x) = \sum_{n=0}^{\infty} \left(\frac{3}{4}\right)^n \phi(4^n x).$$