

Section 4.4

1.) $y = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + \frac{1}{3}$, Domain: all x -values,
 $y' = x^2 - x - 2 = (x-2)(x+1) = 0 \rightarrow x = -1, x = 2$

	+	0	-	0	+	
	-----					y'
rel.	{ $x = -1$		{ $x = 2$		rel.	
max.	{ $y = \frac{3}{2}$		{ $y = -3$		min.	

$y'' = 2x - 1 = 0 \rightarrow x = \frac{1}{2}$

	-	0	+	
	-----			y''
infl.	{ $x = \frac{1}{2}$			
pt.	{ $y = -\frac{3}{4}$			

y is \uparrow for $x < -1, x > 2$;

y is \downarrow for $-1 < x < 2$;

y is \cup for $x > \frac{1}{2}$;

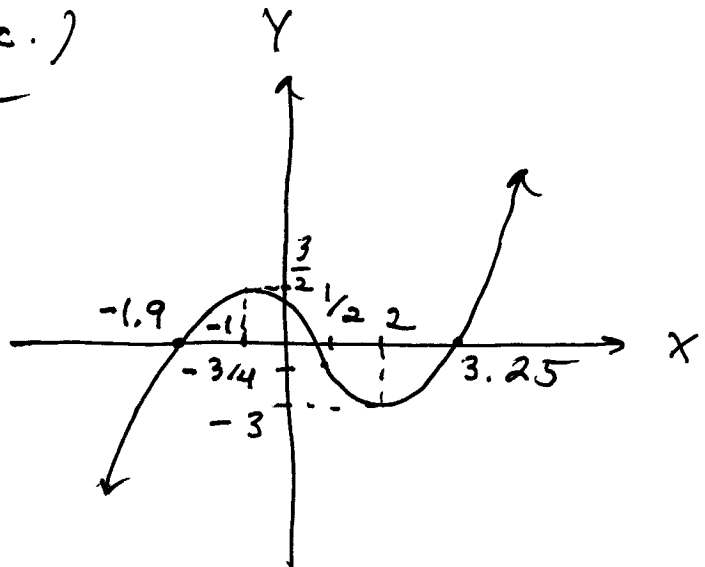
y is \cap for $x < \frac{1}{2}$;

$x = 0: y = \frac{1}{3}$

$y = 0: \frac{x^3}{3} - \frac{x^2}{2} - 2x + \frac{1}{3} = 0$

(use graphing calc.)

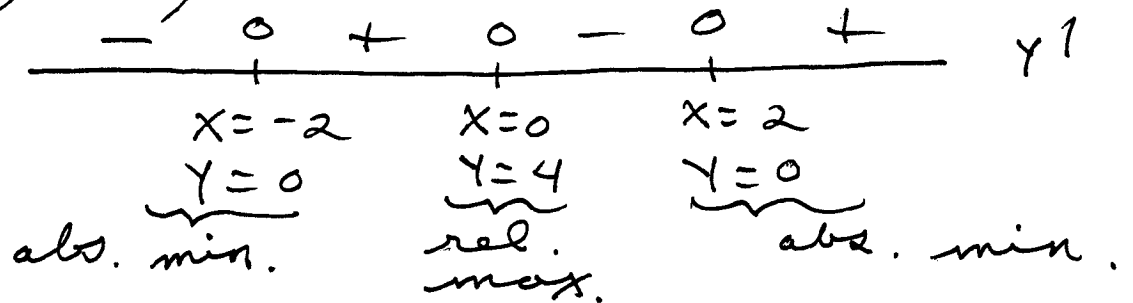
$x \approx -1.9, 0.15, 3.25$



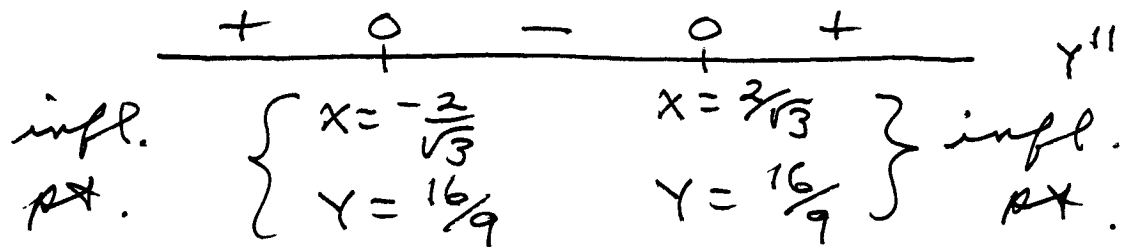
2.) $Y = \frac{x^4}{4} - 2x^2 + 4$, Domain: all x -values,

$$Y' = x^3 - 4x = x(x-2)(x+2) = 0 \rightarrow$$

$$x=0, x=2, x=-2$$



$$Y'' = 3x^2 - 4 = 0 \rightarrow x = \pm \frac{2}{\sqrt{3}}$$



Y is \uparrow for $-2 < x < 0$, $x > 2$;

Y is \downarrow for $x < -2$, $0 < x < 2$;

Y is \cup for $x < -\frac{2}{\sqrt{3}}$, $x > \frac{2}{\sqrt{3}}$;

Y is \cap for $-\frac{2}{\sqrt{3}} < x < \frac{2}{\sqrt{3}}$

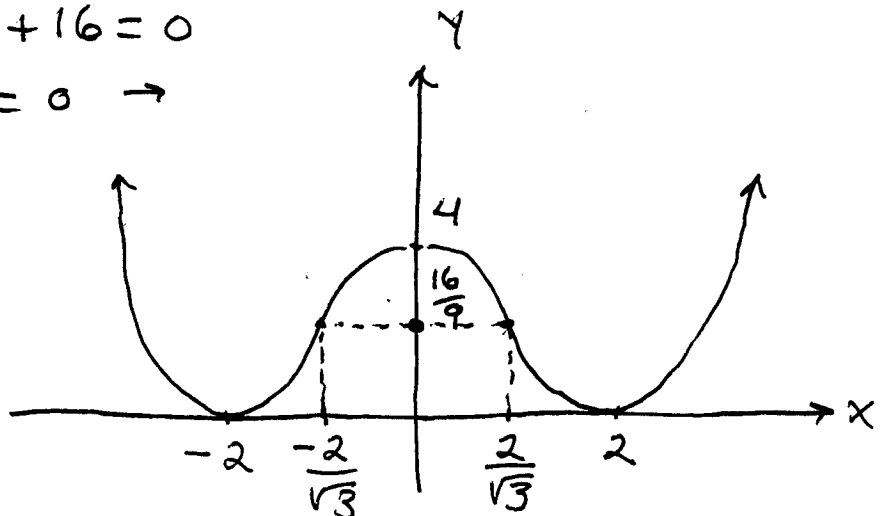
$$x=0: Y=4$$

$$Y=0: \frac{1}{4}(x^2)^2 - 2(x^2) + 4 = 0$$

$$\rightarrow (x^2)^2 - 8(x^2) + 16 = 0$$

$$(x^2 - 4)(x^2 + 4) = 0 \rightarrow$$

$$x = 2, x = -2$$



10.) $Y = 6 - 2X - X^2$, Domain: all x -values,

$$Y' = -2 - 2X = -2(1+X) = 0 \rightarrow X = -1$$

+	0	-	Y'
abs.	$\begin{cases} X = -1 \\ Y = 7 \end{cases}$		
max.			

$$Y'' = -2$$

-	-	-	Y''

Y is \uparrow for $X < -1$;
 Y is \downarrow for $X > -1$;
 Y is \wedge for all x -values

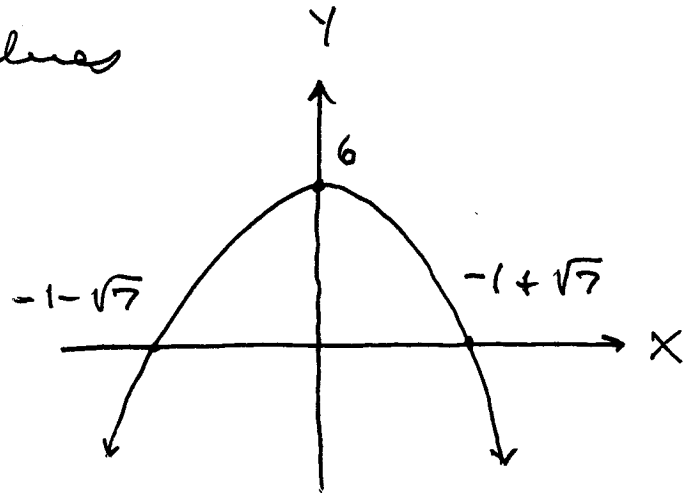
$$X=0: Y=6$$

$$Y=0: X^2 + 2X - 6 = 0$$

$$\rightarrow X = \frac{-2 \pm \sqrt{4 - (-24)}}{2}$$

$$= \frac{-2 \pm 2\sqrt{7}}{2}$$

$$= -1 \pm \sqrt{7}$$



15.) $Y = X^4 - 2X^2$, Domain: all x -values,

$$Y' = 4X^3 - 4X = 4X(X-1)(X+1) = 0 \rightarrow$$

$$X=0, X=1, X=-1$$

-	0	+	0	-	0	+	Y'
X = -1	X = 0	X = 1					
Y = -1	Y = 0	Y = -1					
<u>Y = -1</u>	<u>Y = 0</u>	<u>Y = -1</u>					
abs. min.	rel. max.	abs. min.					

$$Y'' = 12x^2 - 4 = 4(3x^2 - 1) = 0 \rightarrow x = \pm \frac{1}{\sqrt{3}}$$

+	0	-	0	+	Y''
infl. pt.	{	$x = -\frac{1}{\sqrt{3}}$		$x = \frac{1}{\sqrt{3}}$	} infl. pt.
		$Y = -\frac{5}{9}$		$Y = \frac{5}{9}$	

Y is \uparrow for $-1 < x < 0, x > 1$;

Y is \downarrow for $x < -1, 0 < x < 1$;

Y is \cup for $x < -\frac{1}{\sqrt{3}}, x > \frac{1}{\sqrt{3}}$;

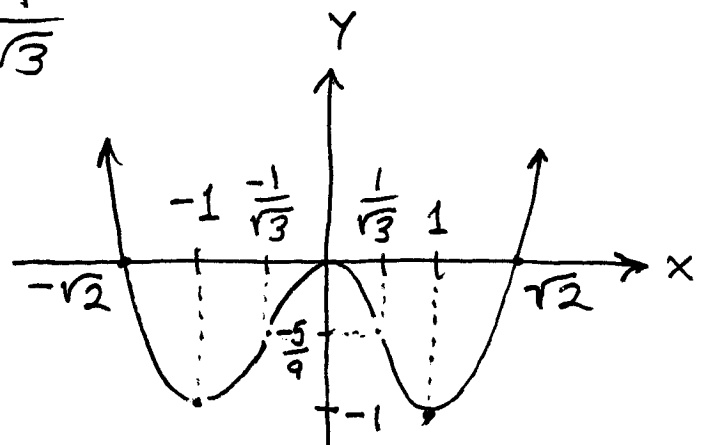
Y is \cap for $-\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$

$$x=0: Y=0$$

$$Y=0: x^4 - 2x^2 = 0$$

$$\rightarrow x^2(x^2 - 2) = 0$$

$$\rightarrow x=0, x = \pm \sqrt{2}$$



19.) $Y = x^5 - 5x^4$ Domain: all x -values,

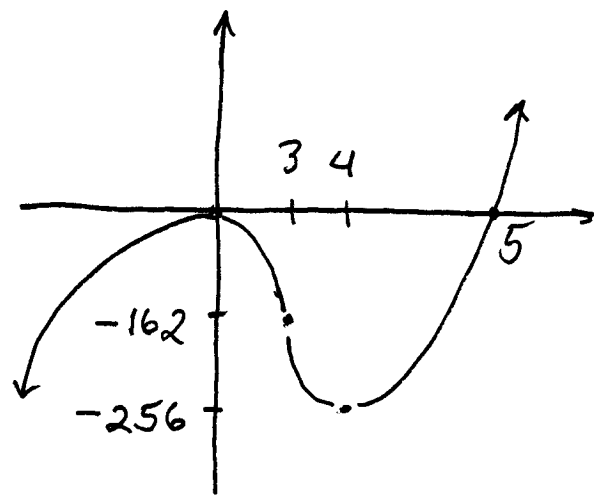
$$Y' = 5x^4 - 20x^3 = 5x^3(x - 4) = 0$$

+	0	-	0	+	Y'
rel. max.	{	$x=0$		$x=4$	} rel. min.
		$Y=0$		$Y=-256$	

$$Y'' = 20x^3 - 60x^2 = 20x^2(x - 3) = 0$$

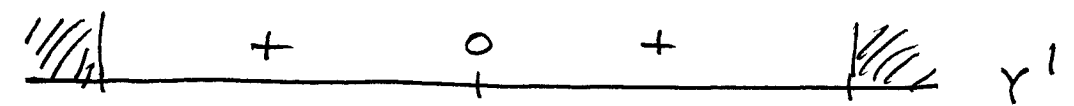
-	0	-	0	+	Y''
	$x=0$		$x=3$		} infl. pt.
			$Y=-162$		}

$$\begin{aligned}
 x=0 &: y=0 \\
 y=0 &: x^5 - 5x^4 = 0 \\
 \rightarrow & x^4(x-5) = 0 \\
 \rightarrow & x=0, x=5
 \end{aligned}$$



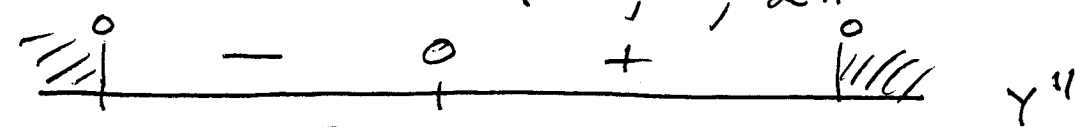
21.) $y = x + \sin x$ $0 \leq x \leq 2\pi$

$$y' = 1 + \cos x = 0 \rightarrow \cos x = -1 \rightarrow x = \pi$$



abs. min. $\left\{ \begin{array}{l} x=0 \\ y=0 \end{array} \right.$ $x=\pi$ $\left. \begin{array}{l} x=2\pi \\ y=2\pi \end{array} \right\}$ abs. max.

$$y'' = -\sin x = 0 \rightarrow x = 0, \pi, 2\pi$$



$x=0$ inf. pt. $\left\{ \begin{array}{l} x=\pi \\ y=\pi \end{array} \right.$ $x=2\pi$

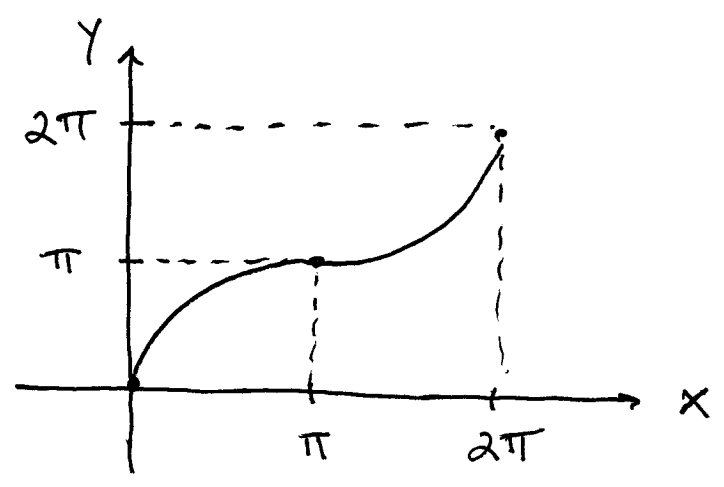
y is \uparrow for $0 < x < \pi$, $\pi < x < 2\pi$;

y is \cup for $\pi < x < 2\pi$;

y is \wedge for $0 < x < \pi$

$$x=0 : y=0$$

$$y=0 : x=0$$

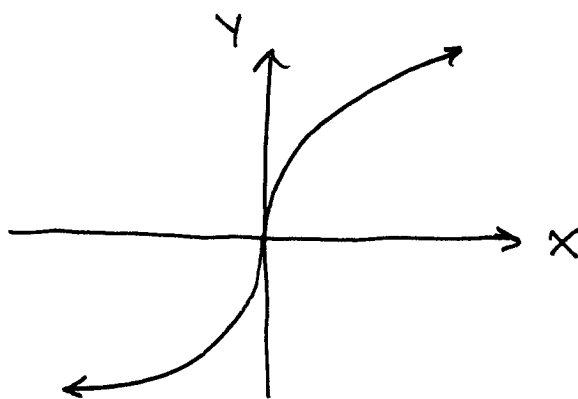
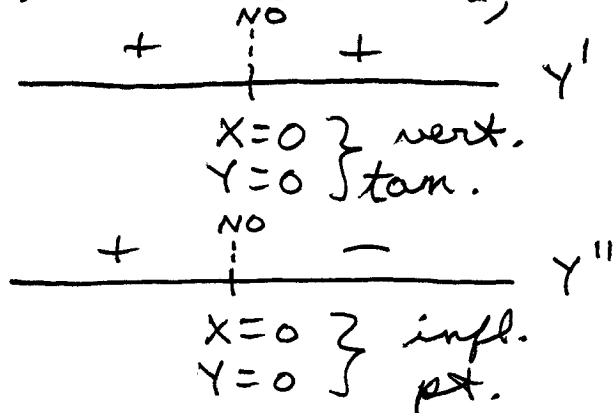


23.) $Y = X^{1/5}$ Domain: all x -values,
 $Y' = \frac{1}{5} X^{-4/5} = \frac{1}{5X^{4/5}}$

$Y'' = \frac{-4}{25} X^{-9/5} = \frac{-4}{25X^{9/5}}$

Y is \uparrow for $x < 0, x > 0$;
 Y is \cup for $x < 0$;
 Y is \cap for $x > 0$;

$x=0 : Y=0$
 $Y=0 : X=0$

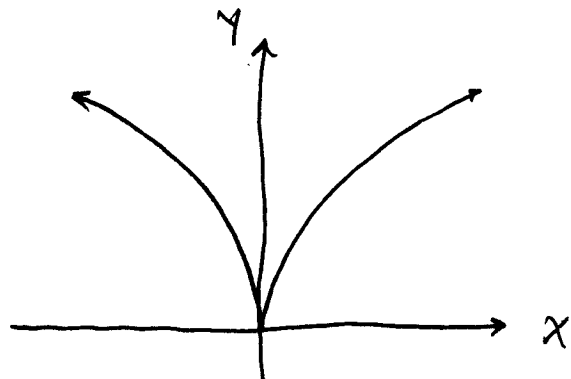
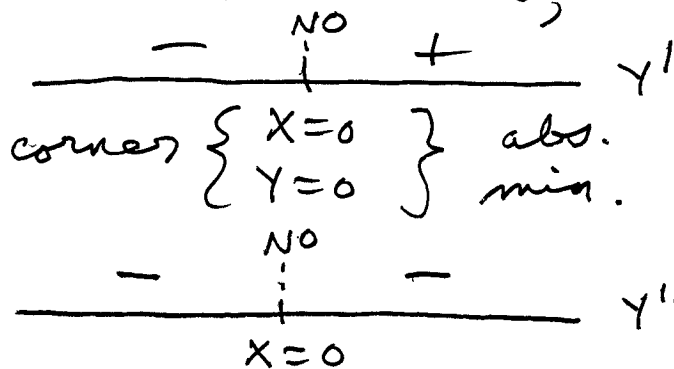


24.) $Y = X^{2/5}$ Domain: all x -values,
 $Y' = \frac{2}{5} X^{-3/5} = \frac{2}{5X^{3/5}}$

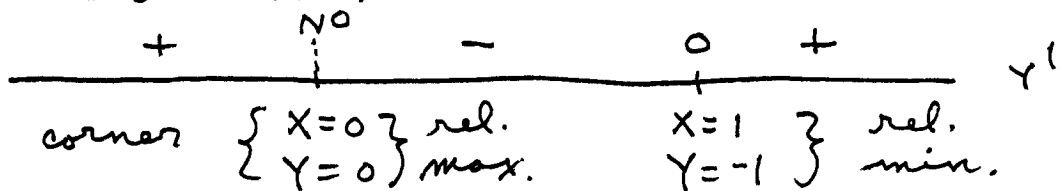
$Y'' = \frac{-6}{25} X^{-8/5} = \frac{-6}{25X^{8/5}}$

Y is \uparrow for $x > 0$,
 Y is \downarrow for $x < 0$,
 Y is \cap for $x < 0, x > 0$,

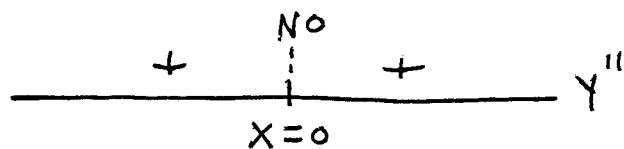
$x=0 : Y=0$
 $Y=0 : X=0$



25.) $Y = 2x - 3x^{2/3}$ Domain: all x -values,
 $Y' = 2 - 3 \cdot \frac{2}{3} x^{-1/3} = 2 - \frac{2}{x^{1/3}} = 2 \left(\frac{x^{1/3} - 1}{x^{1/3}} \right) = 0$
 $\rightarrow x^{1/3} - 1 = 0 \rightarrow x = 1$



$Y'' = -2 \cdot \frac{1}{3} x^{-4/3} = \frac{2}{3x^{4/3}} = 0$



Y is \uparrow for $x < 0, x > 1$,
 Y is \downarrow for $0 < x < 1$,
 Y is \cup for $x < 0, x > 0$;

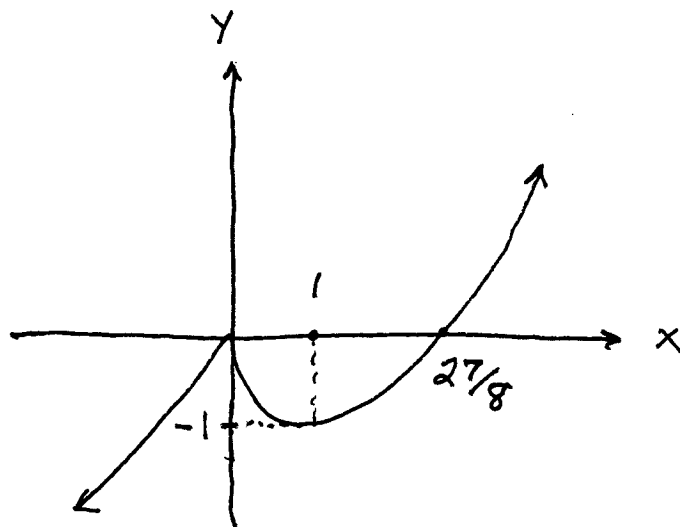
$X=0: Y=0$

$Y=0: 2x - 3x^{2/3} = 0$

$\rightarrow x^{2/3} (2x^{1/3} - 3) = 0$

$\rightarrow x = 0, x^{1/3} = 3/2$

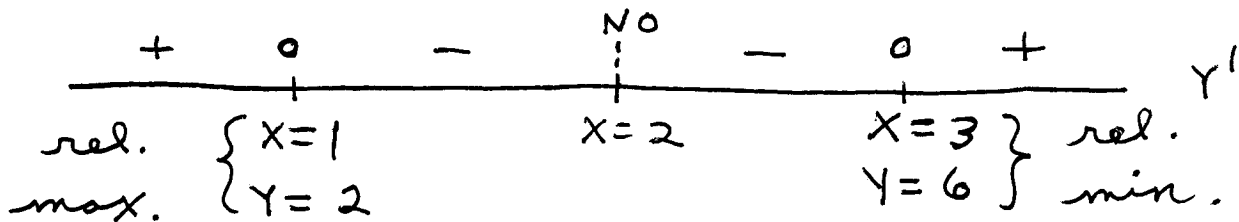
$\rightarrow x = 27/8$



29.) $Y = \frac{x^2 - 3}{x - 2}$, Domain: all $x \neq 2$,

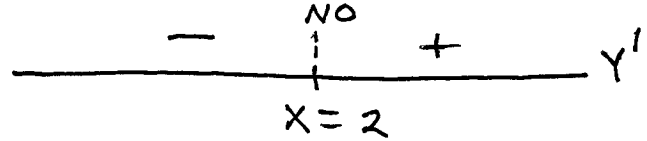
$Y' = \frac{(x-2) \cdot 2x - (x^2-3)(1)}{(x-2)^2} = \frac{2x^2 - 4x - x^2 + 3}{(x-2)^2}$

$= \frac{x^2 - 4x + 3}{(x-2)^2} = \frac{(x-3)(x-1)}{(x-2)^2} = 0$



$$Y'' = \frac{(x-2)^2 \cdot (2x-4) - (x^2-4x+3) \cdot 2(x-2)}{(x-2)^2}$$

$$= \frac{2(x-2) [x^2 - 4x + 4 - x^2 + 4x - 3]}{(x-2)^2} = \frac{2}{x-2} = 0$$



$$x=0: Y = 3/2$$

$$Y=0: x^2-3=0 \rightarrow x = \pm\sqrt{3}$$

$$\lim_{x \rightarrow \pm\infty} \frac{x^2-3}{x-2} \cdot \frac{1/x}{1/x} = \lim_{x \rightarrow \pm\infty} \frac{x - 3/x}{1 - 2/x} = \pm\infty,$$

$$\lim_{x \rightarrow 2^+} \frac{x^2-3}{x-2} = \frac{1}{0^+} = +\infty,$$

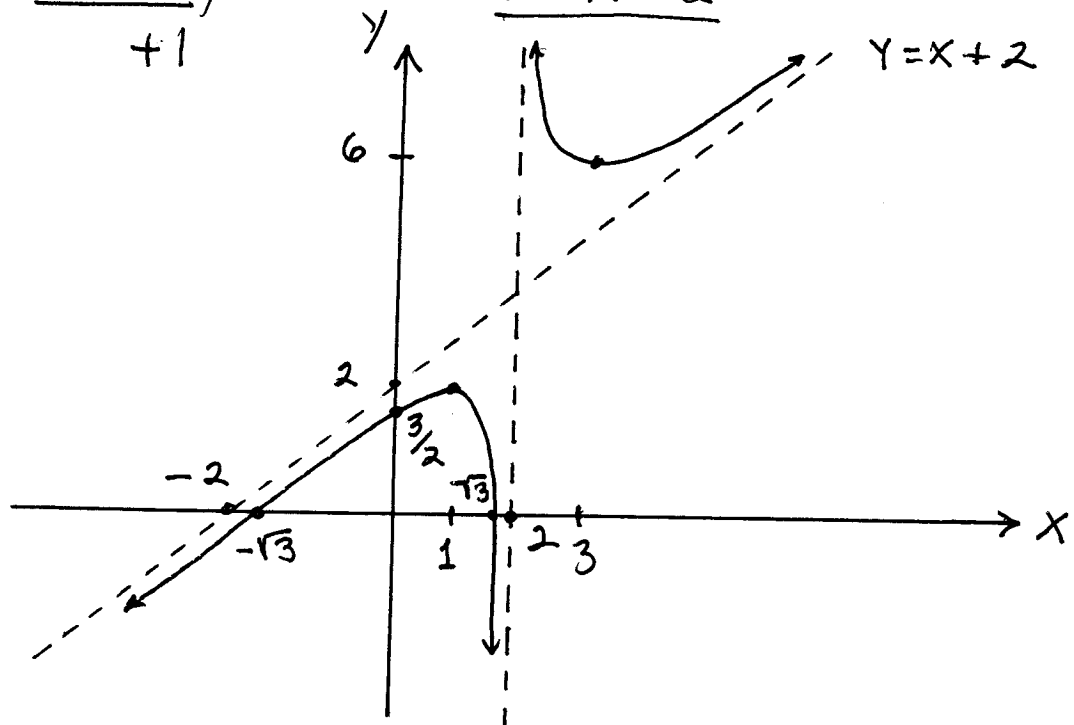
$$\lim_{x \rightarrow 2^-} \frac{x^2-3}{x-2} = \frac{1}{0^-} = -\infty, \quad \underline{\text{V.A. : } x=2}$$

$$\begin{array}{r} x+2 \\ x-2 \overline{) x^2-3} \\ \underline{-(x^2-2x)} \\ 2x-3 \\ \underline{-(2x-4)} \\ +1 \end{array}$$

$$\text{so } \frac{x^2-3}{x-2} = x+2 + \frac{1}{x-2}$$

so tilted asymptote is

$$Y = X + 2$$



36.) $Y = x (\ln x)^2$, Domain: $x > 0$,
 $Y' = x \cdot 2 \ln x \cdot \frac{1}{x} + (\ln x)^2 = \ln x (2 + \ln x) = 0$

$\begin{array}{cccccc} \text{///} & & & & & \\ | & + & 0 & - & 0 & + \\ \hline & & & & & Y' \end{array}$

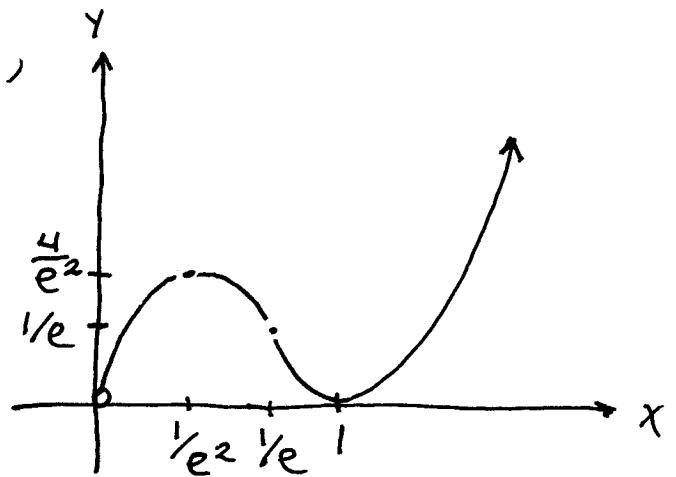
$x=0$ $\left\{ \begin{array}{l} x=e^{-2} \\ Y=4e^{-2} \end{array} \right.$ $\left. \begin{array}{l} x=1 \\ Y=0 \end{array} \right\}$ abs. min.
 rel. max.

$Y'' = 2 \cdot \frac{1}{x} + 2 \ln x \cdot \frac{1}{x} = 2 \cdot \frac{1}{x} (1 + \ln x) = 0$

$\begin{array}{cccc} \text{///} & - & 0 & + \\ | & & & \\ \hline & & & Y'' \end{array}$

$x=0$ $\left. \begin{array}{l} x=e^{-1} \\ Y=e^{-1} \end{array} \right\}$ infl. pt.

Y is \uparrow for $0 < x < \frac{1}{e^2}$, $x > 1$,
 Y is \downarrow for $\frac{1}{e^2} < x < 1$,
 Y is \cup for $x > \frac{1}{e}$,
 Y is \cap for $0 < x < \frac{1}{e}$;
 $x=0$: (No)
 $Y=0$: $x=1$



42.) $Y = \frac{e^x}{1+e^x}$, Domain: all x -values,

$Y' = \frac{(1+e^x) \cdot e^x - e^x \cdot e^x}{(1+e^x)^2} = \frac{e^x}{(1+e^x)^2} = 0$

$\begin{array}{cccc} & + & + & + \\ \hline & & & Y' \end{array}$

$Y'' = \frac{(1+e^x)^2 \cdot e^x - e^x \cdot 2(1+e^x) \cdot e^x}{(1+e^x)^4}$

$$= \frac{e^x(1+e^x) \cdot [1+e^x - 2e^x]}{(1+e^x)^3} = \frac{e^x(1-e^x)}{(1+e^x)^3} = 0$$

$$\rightarrow 1-e^x = 0 \rightarrow e^x = 1 \rightarrow x = 0$$

$$\begin{array}{c} + \quad \quad 0 \quad \quad - \\ \hline \end{array} \quad Y''$$

$x=0 \quad \left. \begin{array}{l} \text{infl.} \\ \text{pt.} \end{array} \right\}$
 $Y = \frac{1}{2}$

Y is \uparrow for all x -values,

Y is \cup for $x < 0$,

Y is \cap for $x > 0$;

$$x=0: Y = \frac{1}{2}$$

$$Y=0: (\text{no})$$

$$\lim_{x \rightarrow \infty} \frac{e^x}{1+e^x} \cdot \frac{\frac{1}{e^x}}{\frac{1}{e^x}} = \lim_{x \rightarrow \infty} \frac{1}{\frac{1}{e^x} + 1} = \frac{1}{0+1} = 1$$

so H.A.: $Y = 1$

$$\lim_{x \rightarrow -\infty} \frac{e^x}{1+e^x} = \frac{e^{-\infty}}{1+e^{-\infty}} = \frac{0}{1+0} = 0$$

so H.A.: $Y = 0$

