

Section 4.6

$$7.) \lim_{x \rightarrow 2} \frac{x-2}{x^2-4} \stackrel{0/0}{=} \lim_{x \rightarrow 2} \frac{1}{2x} = \frac{1}{4}$$

$$10.) \lim_{t \rightarrow 1} \frac{t^3-1}{4t^3-t-3} \stackrel{0/0}{=} \lim_{t \rightarrow 1} \frac{3t^2}{12t^2-1} = \frac{3}{11}$$

$$13.) \lim_{t \rightarrow 0} \frac{\sin t^2}{t} \stackrel{0/0}{=} \lim_{t \rightarrow 0} \frac{\cos t^2 \cdot 2t}{1} = (1)(0) = 0$$

$$14.) \lim_{t \rightarrow 0} \frac{\sin 5t}{2t} \stackrel{0/0}{=} \lim_{t \rightarrow 0} \frac{\cos 5t \cdot 5}{2} = \frac{(1) \cdot 5}{2} = \frac{5}{2}$$

$$15.) \lim_{x \rightarrow 0} \frac{8x^2}{\cos x - 1} \stackrel{0/0}{=} \lim_{x \rightarrow 0} \frac{16x}{-\sin x}$$

$$\stackrel{0/0}{=} \lim_{x \rightarrow 0} \frac{16}{-\cos x} = \frac{16}{-1} = -16$$

$$18.) \lim_{\theta \rightarrow -\pi/3} \frac{3\theta + \pi}{\sin(\theta + \pi/3)} \stackrel{0/0}{=} \lim_{\theta \rightarrow -\pi/3} \frac{3}{\cos(\theta + \pi/3)} = \frac{3}{1} = 3$$

$$20.) \lim_{x \rightarrow 1} \frac{x-1}{\ln x - \sin \pi x} \stackrel{0/0}{=} \lim_{x \rightarrow 1} \frac{1}{\frac{1}{x} - \pi \cos \pi x}$$

$$= \frac{1}{1 - \pi(-1)} = \frac{1}{1 + \pi}$$

$$21.) \lim_{x \rightarrow 0} \frac{x^2}{\ln(\sec x)} \stackrel{0/0}{=} \lim_{x \rightarrow 0} \frac{2x}{\frac{1}{\sec x} \cdot \sec x \tan x}$$

$$= \lim_{x \rightarrow 0} \frac{2x}{\tan x} \stackrel{0/0}{=} \lim_{x \rightarrow 0} \frac{2}{\sec^2 x} = \frac{2}{(1)^2} = 2$$

$$24.) \lim_{t \rightarrow 0} \frac{t \sin t}{1 - \cos t} \stackrel{0/0}{=} \lim_{t \rightarrow 0} \frac{t \cos t + \sin t}{\sin t}$$

$$\begin{aligned} & \stackrel{\text{"0/0"}}{=} \lim_{t \rightarrow 0} \frac{t \cdot -\sin t + \cos t + \cos t}{\cos t} \\ & = \frac{0 + 1 + 1}{1} = 2 \end{aligned}$$

$$26.) \lim_{x \rightarrow \frac{\pi}{2}^-} \left(\frac{\pi}{2} - x\right) \tan x = \text{"0} \cdot \infty \text{"}$$

$$= \lim_{x \rightarrow \frac{\pi}{2}^-} \frac{\frac{\pi}{2} - x}{\cot x} \stackrel{\text{"0/0"}}{=} \lim_{x \rightarrow \frac{\pi}{2}^-} \frac{-1}{-\csc^2 x}$$

$$= \lim_{x \rightarrow \frac{\pi}{2}^-} \sin^2 x = (1)^2 = 1$$

$$27.) \lim_{\theta \rightarrow 0} 3 \frac{\sin \theta}{-1} \stackrel{\text{"0/0"}}{=} \lim_{\theta \rightarrow 0} \frac{\sin \theta}{3 \cdot \cos \theta \cdot \ln 3}$$

$$= (1)(1) \ln 3 = \ln 3$$

$$29.) \lim_{x \rightarrow 0} \frac{x \cdot 2^x}{2^x - 1} \stackrel{\text{"0/0"}}{=} \lim_{x \rightarrow 0} \frac{x \cdot 2^x \ln 2 + 2^x}{2^x \ln 2}$$

$$= \frac{0+1}{(1) \ln 2} = \frac{1}{\ln 2}$$

$$32.) \lim_{x \rightarrow \infty} \frac{\log_2 x}{\log_3 (x+3)} \stackrel{\text{"}\infty/\infty\text{"}}{=} \lim_{x \rightarrow \infty} \frac{\frac{1}{x} \cdot \frac{1}{\ln 2}}{\frac{1}{x+3} \cdot \frac{1}{\ln 3}}$$

$$= \lim_{x \rightarrow \infty} \frac{x+3}{x} \cdot \frac{\ln 3}{\ln 2} = \lim_{x \rightarrow \infty} \left(1 + \frac{3}{x}\right) \cdot \frac{\ln 3}{\ln 2}$$

$$= (1+0) \frac{\ln 3}{\ln 2} = \frac{\ln 3}{\ln 2}$$

$$\begin{aligned}
 34.) \lim_{x \rightarrow 0^+} \frac{\ln(e^x - 1)}{\ln x} &\stackrel{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x \rightarrow 0^+} \frac{\frac{1}{e^x - 1} \cdot e^x}{\frac{1}{x}} \\
 &= \lim_{x \rightarrow 0^+} \frac{x e^x}{e^x - 1} \stackrel{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x \rightarrow 0^+} \frac{x e^x + e^x}{e^x} = \frac{1}{1} = 1
 \end{aligned}$$

$$\begin{aligned}
 35.) \lim_{Y \rightarrow 0} \frac{\sqrt{5Y+25} - 5}{Y} &\stackrel{\text{"}\frac{0}{0}\text{"}}{=} \lim_{Y \rightarrow 0} \frac{\frac{1}{2}(5Y+25)^{-\frac{1}{2}} \cdot 5}{1} \\
 &= \lim_{Y \rightarrow 0} \frac{5}{2\sqrt{5Y+25}} = \frac{5}{2(5)} = \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 41.) \lim_{x \rightarrow 1^+} \left(\frac{1}{x-1} - \frac{1}{\ln x} \right) &= \text{"}\infty - \infty\text{"} \\
 &= \lim_{x \rightarrow 1^+} \frac{\ln x - (x-1)}{(x-1)\ln x} \stackrel{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x \rightarrow 1^+} \frac{\frac{1}{x} - 1}{(x-1) \cdot \frac{1}{x} + (1)\ln x} \cdot \frac{x}{x} \\
 &= \lim_{x \rightarrow 1^+} \frac{1-x}{x-1+x\ln x} \stackrel{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x \rightarrow 1^+} \frac{-1}{1+x \cdot \frac{1}{x} + (1)\ln x} \\
 &= \lim_{x \rightarrow 1^+} \frac{-1}{1+1+\ln x} = \frac{-1}{2+0} = -\frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 44.) \lim_{h \rightarrow 0} \frac{e^h - 1 - h}{h^2} &\stackrel{\text{"}\frac{0}{0}\text{"}}{=} \lim_{h \rightarrow 0} \frac{e^h - 1}{2h} \\
 &\stackrel{\text{"}\frac{0}{0}\text{"}}{=} \lim_{h \rightarrow 0} \frac{e^h}{2} = \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 46.) \lim_{x \rightarrow \infty} \frac{x^2}{e^x} &\stackrel{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{x \rightarrow \infty} \frac{2x}{e^x} \stackrel{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{x \rightarrow \infty} \frac{2}{e^x} \\
 &= \frac{2}{\infty} = 0
 \end{aligned}$$

$$49.) \lim_{x \rightarrow \infty} (\ln x)^{1/x} = " \infty^0 "$$

$$= \lim_{x \rightarrow \infty} e^{\ln (\ln x)^{1/x}} = \lim_{x \rightarrow \infty} e^{\frac{1}{x} \ln (\ln x)}$$

$$= \lim_{x \rightarrow \infty} e^{\frac{\ln (\ln x)}{x}} = e^{\lim_{x \rightarrow \infty} \frac{\ln (\ln x)}{x}}$$

$$\begin{matrix} "0/0" \\ "0/0" \\ "0/0" \end{matrix} e^{\lim_{x \rightarrow \infty} \frac{\frac{1}{\ln x} \cdot \frac{1}{x}}{1}} = e^0 = 1$$

$$52.) \lim_{x \rightarrow \infty} x^{1/\ln x} = " \infty^0 "$$

$$= \lim_{x \rightarrow \infty} e^{\ln x^{1/\ln x}} = \lim_{x \rightarrow \infty} e^{\frac{1}{\ln x} \cdot \ln x}$$

$$= \lim_{x \rightarrow \infty} e^1 = e$$

$$53.) \lim_{x \rightarrow \infty} (1+2x)^{\frac{1}{2 \ln x}} = " \infty^0 "$$

$$= \lim_{x \rightarrow \infty} e^{\ln (1+2x)^{\frac{1}{2 \ln x}}} = \lim_{x \rightarrow \infty} e^{\frac{1}{2 \ln x} \cdot \ln (1+2x)}$$

$$= e^{\lim_{x \rightarrow \infty} \frac{\ln (1+2x)}{2 \ln x}} \begin{matrix} " \infty / \infty " \\ " \infty / \infty " \end{matrix} e^{\lim_{x \rightarrow \infty} \frac{1}{2} \cdot \frac{2}{1+2x}}$$

$$= e^{\lim_{x \rightarrow \infty} \frac{x}{1+2x}} \begin{matrix} " \infty / \infty " \\ " \infty / \infty " \end{matrix} e^{\lim_{x \rightarrow \infty} \frac{1}{2}} = e^{1/2}$$

$$54.) \lim_{x \rightarrow 0} (e^x + x)^{1/x} = " 1^{\pm \infty } "$$

$$\begin{aligned}
 &= \lim_{x \rightarrow 0} e^{\ln(e^x + x)^{\frac{1}{x}}} = \lim_{x \rightarrow 0} e^{\frac{1}{x} \ln(e^x + x)} \\
 &= e^{\lim_{x \rightarrow 0} \frac{\ln(e^x + x)}{x}} \stackrel{\text{"0/0"}}{=} e^{\lim_{x \rightarrow 0} \frac{\frac{1}{e^x + x} \cdot (e^x + 1)}{1}} \\
 &= e^{\frac{2}{1+0}} = e^2
 \end{aligned}$$

$$\begin{aligned}
 55.) \lim_{x \rightarrow 0^+} x^x &= \text{"0}^0\text{"} = \lim_{x \rightarrow 0^+} e^{\ln x^x} \\
 &= \lim_{x \rightarrow 0^+} e^{x \ln x} \stackrel{\text{"0} \cdot \infty\text{"}}{=} e^{\lim_{x \rightarrow 0^+} \frac{\ln x}{1/x}}
 \end{aligned}$$

$$\stackrel{\text{"}\infty\text{/}\infty\text{"}}{=} e^{\lim_{x \rightarrow 0^+} \frac{\frac{1}{x}}{-1/x^2}} = e^{\lim_{x \rightarrow 0^+} (-x)} = e^0 = 1$$

$$56.) \lim_{x \rightarrow 0^+} \left(1 + \frac{1}{x}\right)^x = \text{"}\infty\text{}^0\text{"}$$

$$\begin{aligned}
 &= \lim_{x \rightarrow 0^+} e^{\ln\left(1 + \frac{1}{x}\right)^x} = \lim_{x \rightarrow 0^+} e^{x \ln\left(1 + \frac{1}{x}\right)} \\
 &\stackrel{\text{"0} \cdot \infty\text{"}}{=} \lim_{x \rightarrow 0^+} e^{\frac{\ln\left(1 + \frac{1}{x}\right)}{1/x}} \stackrel{\text{"}\infty\text{/}\infty\text{"}}{=} e^{\lim_{x \rightarrow 0^+} \frac{\frac{1}{1 + 1/x} \cdot \frac{-1}{x^2}}{-1/x^2}} \\
 &= e^{\frac{1}{\infty}} = e^0 = 1
 \end{aligned}$$

$$65.) \quad f(x) = \begin{cases} \frac{9x - 3 \sin 3x}{5x^3}, & \text{if } x \neq 0 \\ c, & \text{if } x = 0 \end{cases}$$

$$\lim_{x \rightarrow 0} \frac{9x - 3 \sin 3x}{5x^3} \stackrel{\frac{0}{0}}{=} \lim_{x \rightarrow 0} \frac{9 - 3 \cdot \cos 3x \cdot 3}{15x^2}$$

$$= \lim_{x \rightarrow 0} \frac{9 - 9 \cos 3x}{15x^2} \stackrel{\frac{0}{0}}{=} \lim_{x \rightarrow 0} \frac{-9 \cdot -\sin 3x \cdot 3}{30x}$$

$$= \lim_{x \rightarrow 0} \frac{27 \sin 3x}{30x} = \lim_{x \rightarrow 0} \frac{9 \sin 3x}{10x}$$

$$\stackrel{\frac{0}{0}}{=} \lim_{x \rightarrow 0} \frac{9 \cdot \cos 3x \cdot 3}{10} = \frac{27(1)}{10} = \frac{27}{10};$$

let $c = \frac{27}{10}$; then

$$i.) \quad f(0) = \frac{27}{10}$$

$$ii.) \quad \lim_{x \rightarrow 0} f(x) = \frac{27}{10}$$

$$iii.) \quad \lim_{x \rightarrow 0} f(x) = f(0)$$

and f is continuous at $x = 0$.