Math 21B
Kouba
Simpson's Rule

__Example__:

Use S_4, Simpson's Rule with $n=4$, to estimate the exact value of $\int_{-5}^{-4} \frac{x+1}{x+3}\,dx$.

$f(x) = \frac{x+1}{x+3}$, $n=4$, $h = \frac{-4-(-5)}{4} = \frac{1}{4}$

-5	$\frac{-19}{4}$	$-\frac{9}{2}$	$\frac{-17}{4}$	-4
x_0	x_1	x_2	x_3	x_4

$$S_4 = \frac{\left(\frac{1}{4}\right)}{3}\left[f(-5) + 4f\left(\frac{-19}{4}\right) + 2f\left(\frac{-9}{2}\right) + 4f\left(\frac{-17}{4}\right) + f(-4)\right]$$

$$= \frac{1}{12}\left[2 + 4\left(\frac{15}{7}\right) + 2\left(\frac{7}{3}\right) + 4\left(\frac{13}{5}\right) + 3\right] \approx 2.3865 \ ;$$

__exact value__: $\int_{-5}^{-4} \frac{x+1}{x+3}\,dx = 1 + \ln 4 \approx 2.3863$;

absolute error $|E_4| = \left| \int_{-5}^{-4} \frac{x+1}{x+3}\,dx - S_4 \right| = 0.0002$

__Question__: What should n be in order that S_n, Simpson's Rule with n subdivisions, estimate the exact value of $\int_{-5}^{-4} \frac{x+1}{x+3}\,dx$ with absolute error at most 0.00001 ? $h = \frac{-4-(-5)}{n} = \frac{1}{n}$

absolute error $|E_n| \le \frac{(b-a)\cdot h^4}{180}\left\{\max_{a \le x \le b} |f^{(4)}(x)|\right\} \rightarrow$

$f(x) = \frac{x+1}{x+3}$, $f'(x) = 2(x+3)^{-2}$, $f''(x) = -4(x+3)^{-3}$, $f'''(x) = 12(x+3)^{-4}$, $f^{(4)}(x) = \frac{-48}{(x+3)^5}$ so

$\max_{-5 \le x \le -4} |f^{(4)}(x)| = \frac{48}{|(-4)+3|^5} = 48$; then

$$|E_n| \le \frac{1}{180\, n^4}\cdot\{48\} = \frac{4}{15 n^4} \le 0.00001 \rightarrow$$

$$n^4 \ge \frac{4}{15\,(0.00001)} \rightarrow n \ge \left[\frac{4}{15\,(0.00001)}\right]^{1/4} \approx 12.7 \rightarrow$$ so use $n = 14$!!