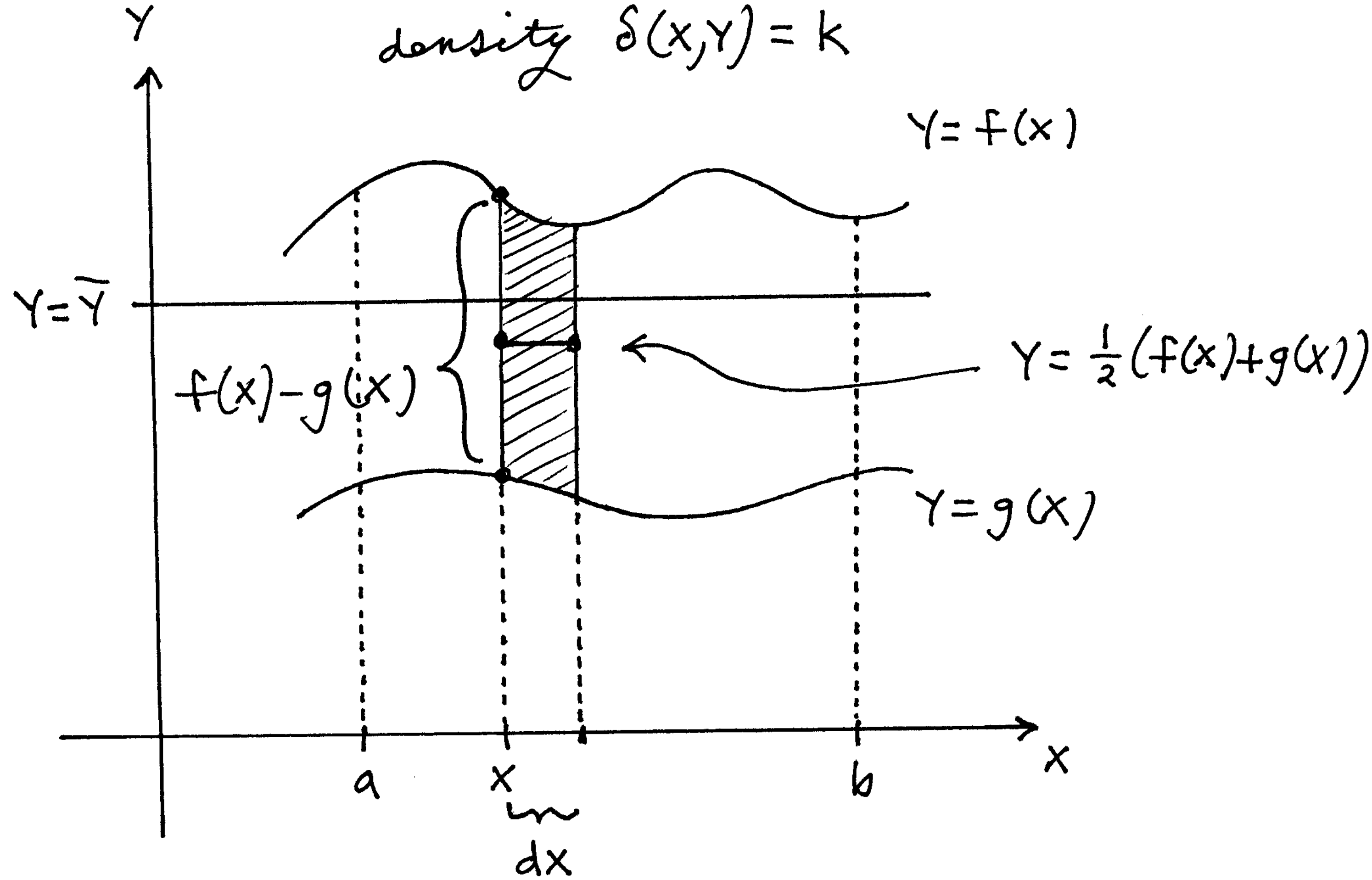
# Alternate Formula for $\bar{Y}$




assume that the center of mass of the thin strip occurs in the middle at $y = \frac{1}{2}(f(x) + g(x))$. Then an estimate for the moment of the thin strip about $y = \bar{y}$ is

$$(\text{mass})(\text{distance}) = (\text{area})(\text{density})(\text{distance})$$

$$\approx (f(x) - g(x)) \cdot dx \cdot k \cdot \left( \frac{1}{2}(f(x) + g(x)) - \bar{y} \right)$$

$$= \left[ \frac{1}{2}(f(x) - g(x))(f(x) + g(x)) - (f(x) - g(x))\bar{y} \right] k\, dx$$

$$= \left[ \tfrac{1}{2}\left((f(x))^2 - (g(x))^2\right) - (f(x) - g(x))\,\bar{Y} \right] k\,dx \;;$$

then total moment of plate about $Y = \bar{Y}$ is

$$\boxed{M_{Y=\bar{Y}} = \int_a^b \left[ \tfrac{1}{2}\left((f(x))^2 - (g(x))^2\right) - (f(x) - g(x))\,\bar{Y} \right] k\,dx} \,.$$

at the center of mass, $M_{Y=\bar{Y}} = 0 \longrightarrow$

$$\int_a^b \left[ \tfrac{1}{2}\left((f(x))^2 - (g(x))^2\right) - (f(x) - g(x))\,\bar{Y} \right] k\,dx = 0 \longrightarrow$$

$$\cancel{k} \int_a^b \tfrac{1}{2}\left((f(x))^2 - (g(x))^2\right) dx = \cancel{k}\,\bar{Y} \int_a^b (f(x) - g(x))\,dx \longrightarrow$$

$$\boxed{\bar{Y} = \frac{\int_a^b \tfrac{1}{2}\left((f(x))^2 - (g(x))^2\right) dx}{\int_a^b (f(x) - g(x))\,dx}}$$