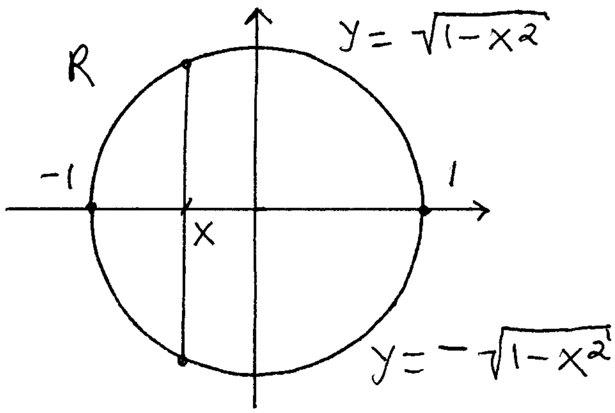
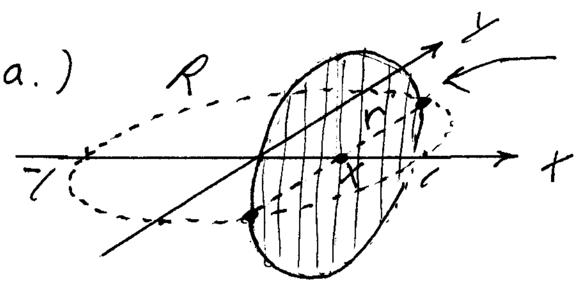
Section 6.1

1.) R



a.)

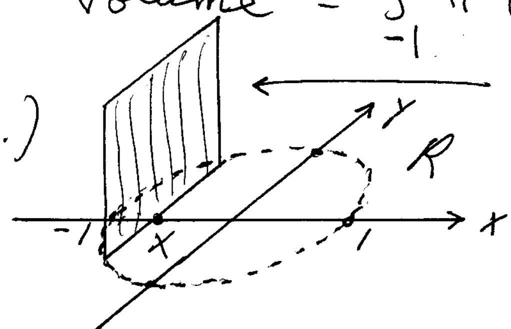


cross-section at x is circle : area

$$A(x) = \pi r^2 = \pi \left(\sqrt{1-x^2}\right)^2 = \pi (1-x^2) ;$$

$$\text{Volume} = \int_{-1}^{1} \pi (1-x^2)\, dx$$

b.)

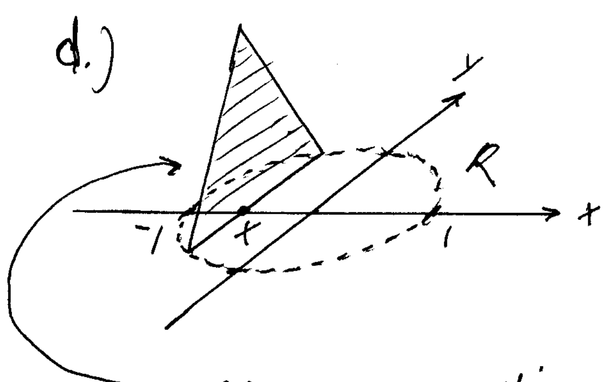


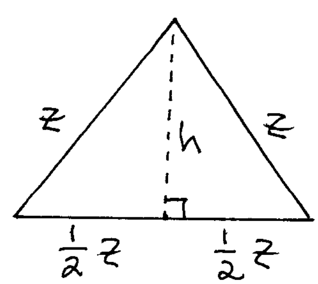
cross-section at x is square : area

$$A(x) = \left(2\sqrt{1-x^2}\right)^2 = 4(1-x^2) ;$$

$$\text{Volume} = \int_{-1}^{1} 4(1-x^2)\, dx$$

d.)





$$h^2 + \left(\tfrac{1}{2}z\right)^2 = z^2 \longrightarrow$$

$$h^2 = \tfrac{3}{4}z^2 \rightarrow h = \tfrac{\sqrt{3}}{2}z ;$$

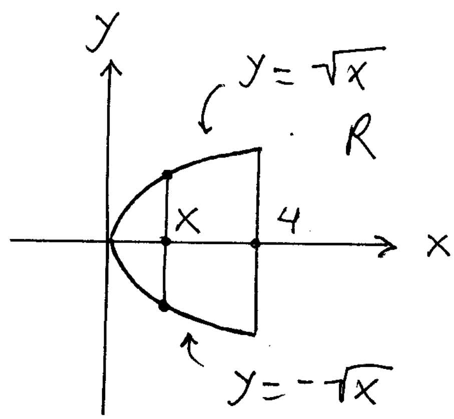
cross-section at x is

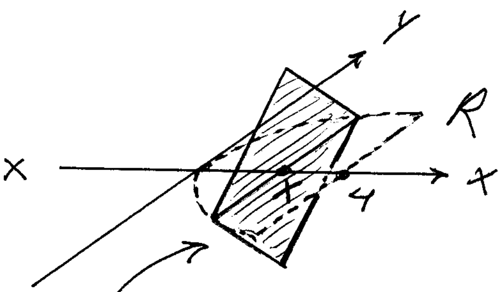
equilateral triangle : area

$$A(x) = \frac{1}{2}(\text{base})(\text{height})$$
$$= \frac{1}{2}\cdot\left(2\sqrt{1-x^2}\right)\cdot\frac{\sqrt{3}}{2}\left(2\sqrt{1-x^2}\right)$$
$$= \sqrt{3}\,(1-x^2) \quad ;$$

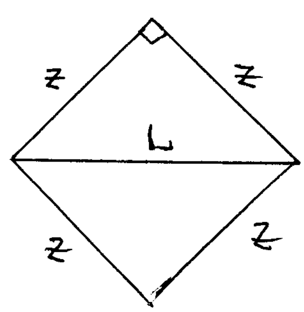
$$\text{Volume} = \int_{-1}^{1}\sqrt{3}\,(1-x^2)\,dx$$

3.)





cross-section at x is square with diagonal in region R : area

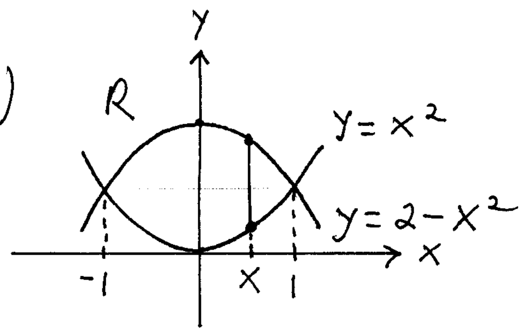


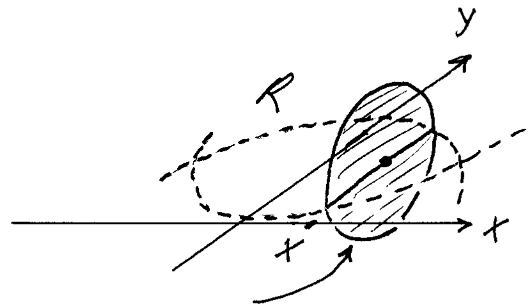
$$z^2 + z^2 = L^2 \rightarrow$$
$$2z^2 = L^2 \rightarrow$$
$$z = \frac{1}{\sqrt{2}}L$$

$$A(x) = (\text{edge})^2$$
$$= \left(\frac{1}{\sqrt{2}}\cdot 2\sqrt{x}\right)^2$$
$$= 2x \quad ;$$

$$\text{Volume} = \int_0^4 2x\,dx$$

4.)



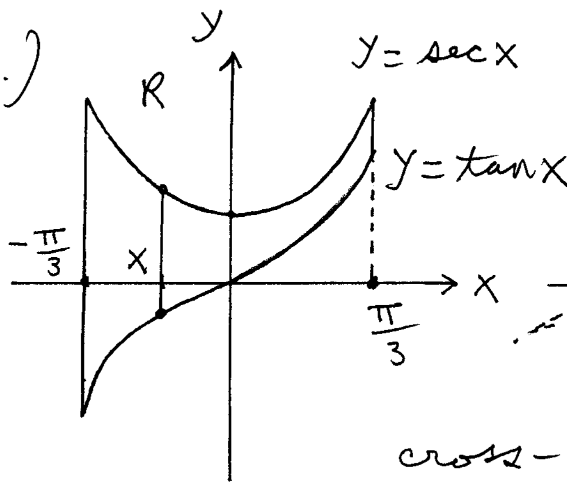


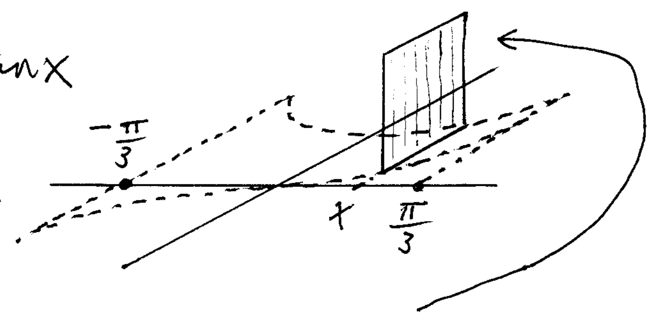
cross-section at x is circle : area

$$A(x) = \pi r^2 = \pi \left(\tfrac{1}{2}\left((2-x^2) - x^2\right)\right)^2$$
$$= \pi \left(\tfrac{1}{2}(2 - 2x^2)\right)^2 = \pi (1 - x^2)^2 \ ;$$

$$\text{Volume} = \int_{-1}^{1} \pi (1 - x^2)^2 \, dx$$

8.) b.)

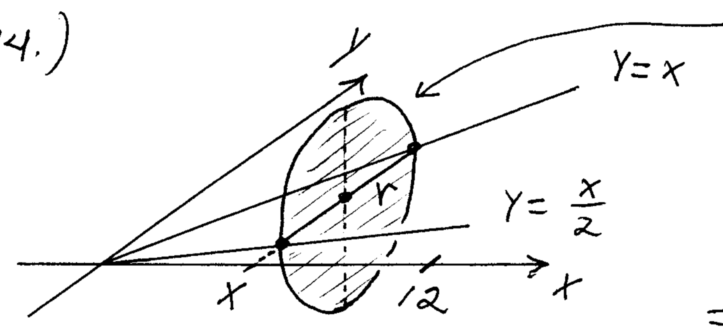




cross-section at x is square : area

$$A(x) = (\text{edge})^2 = (\sec x - \tan x)^2 \ ;$$
$$\text{Volume} = \int_{-\pi/3}^{\pi/3} (\sec x - \tan x)^2 \, dx$$

14.)



cross-section at x is circle: area

$$A(x) = \pi r^2$$
$$= \pi \left(\tfrac{1}{2}\left(x - \tfrac{1}{2}x\right)\right)^2$$
$$= \pi \cdot \left(\tfrac{1}{4}x\right)^2$$
$$= \frac{\pi}{16} x^2 \ ;$$

$$\text{Volume} = \int_0^{12} \frac{\pi}{16} x^2\,dx = \frac{\pi}{16}\cdot \frac{x^3}{3}\Big|_0^{12}$$

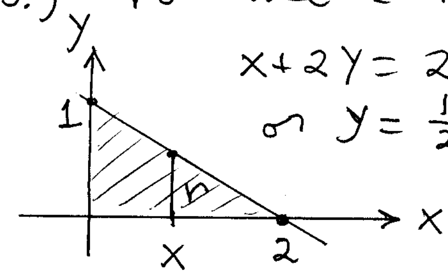
$$= \frac{\pi}{16}\frac{(12)^3}{3} = 36\pi \; ;$$

Volume of right circular cylinder is

$$\text{Volume} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi (3)^2 (12) = 36\pi$$

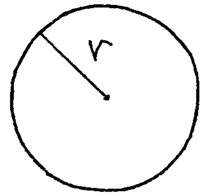
15.) $\text{Volume} = \pi \int_0^2 (\text{radius})^2\,dx$



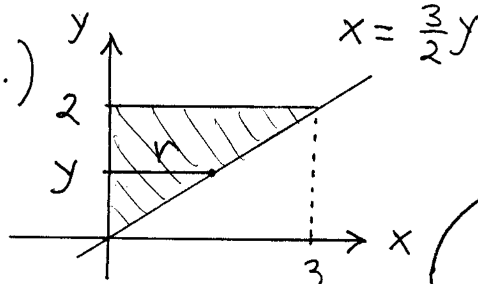
$x + 2y = 2$

or $y = \frac{1}{2}(2-x)$

$$= \pi \int_0^2 \left(\frac{1}{2}(2-x)\right)^2 dx$$

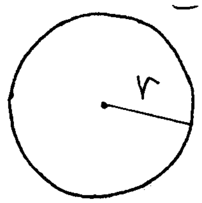


16.)

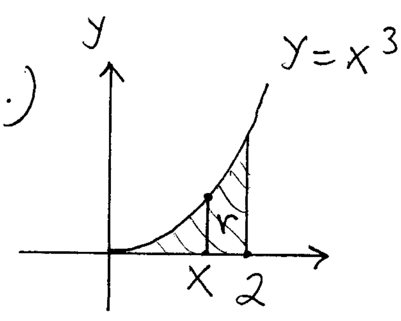


$$\text{Volume} = \pi \int_0^2 (\text{radius})^2\,dy$$

$$= \pi \int_0^2 \left(\frac{3}{2}y\right)^2 dy$$

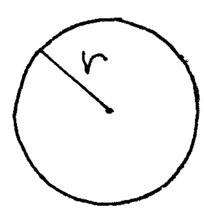


20.)

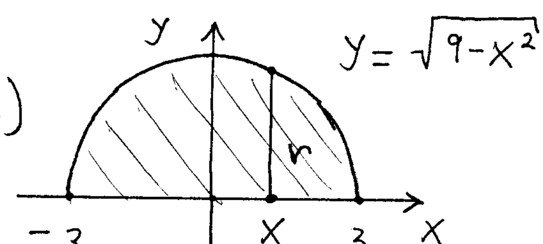


$$\text{Volume} = \pi \int_0^2 (\text{radius})^2\,dx$$

$$= \pi \int_0^2 (x^3)^2\,dx$$

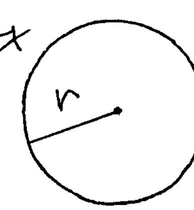


21.)

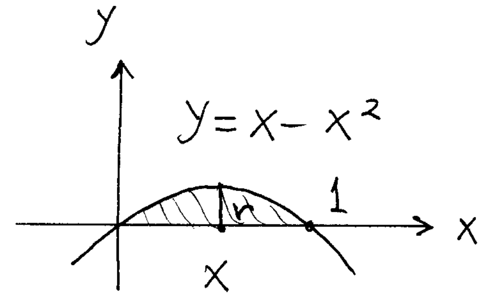


$$\text{Volume} = \pi \int_{-3}^{3} (\text{radius})^2\,dx$$

$$= \pi \int_{-3}^{3} \left(\sqrt{9-x^2}\right)^2 dx$$



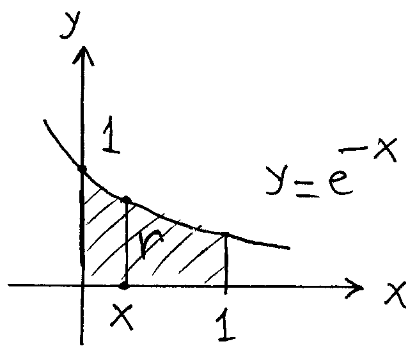
22.) $y = x - x^2 = x(1-x)$



$$\text{Volume} = \pi \int_0^1 (\text{radius})^2\,dx$$

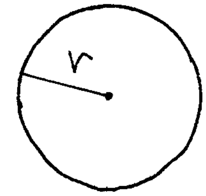
$$= \pi \int_0^1 (x - x^2)^2\,dx$$

25.)

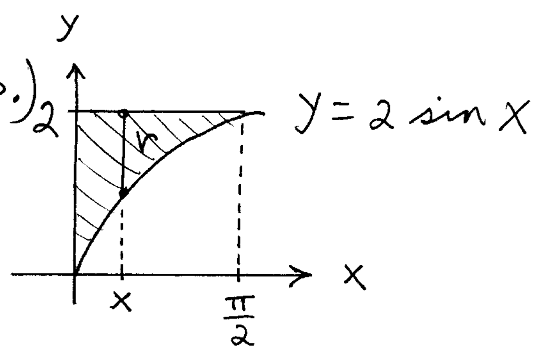


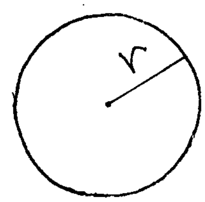
$$\text{Volume} = \pi \int_0^1 (\text{radius})^2\,dx$$

$$= \pi \int_0^1 (e^{-x})^2\,dx$$



30.)

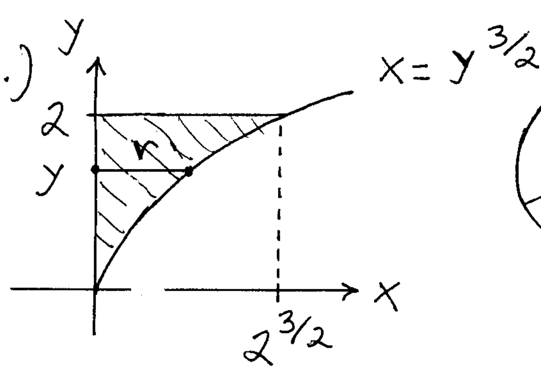


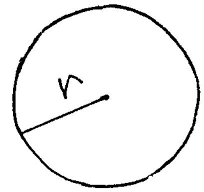


$$\text{Volume} = \pi \int_0^{\frac{\pi}{2}} (\text{radius})^2\,dx$$

$$= \pi \int_0^{\frac{\pi}{2}} (2 - 2\sin x)^2\,dx$$

32.)

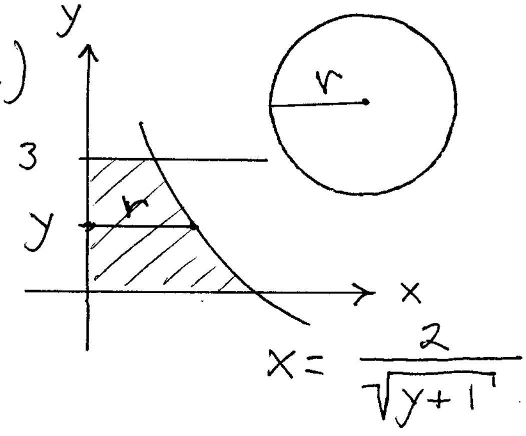




$$\text{Volume} = \pi \int_0^2 (\text{radius})^2\,dy$$

$$= \pi \int_0^2 (y^{3/2})^2\,dy$$

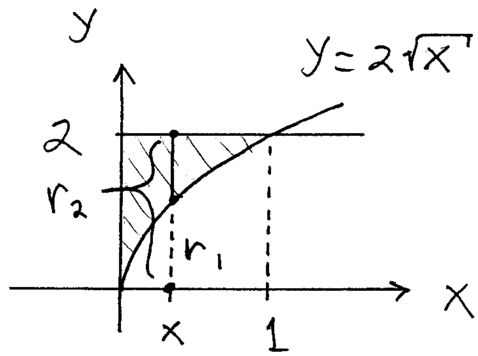
35.)

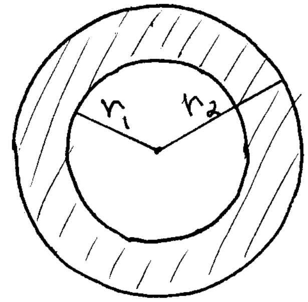


$$\text{Volume} = \pi \int_0^3 (\text{radius})^2\,dy$$

$$= \pi \int_0^3 \left(\frac{2}{\sqrt{y+1}}\right)^2\,dy$$

40.)

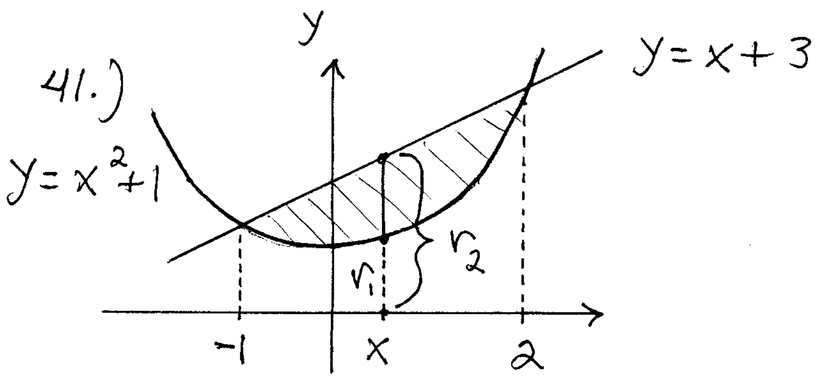




$$\text{Volume} = \pi \int_0^1 (r_2)^2\,dx - \pi \int_0^1 (r_1)^2\,dx$$

$$= \pi \int_0^1 (2)^2\,dx - \pi \int_0^1 (2\sqrt{x})^2\,dx$$

41.)



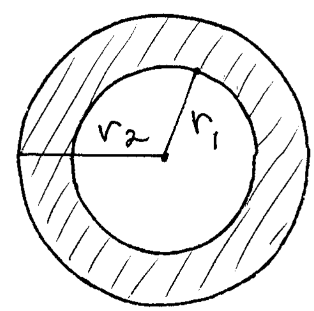
$x^2+1 = x+3 \rightarrow$

$x^2 - x - 2 = 0 \rightarrow$

$(x-2)(x+1) = 0 \rightarrow$

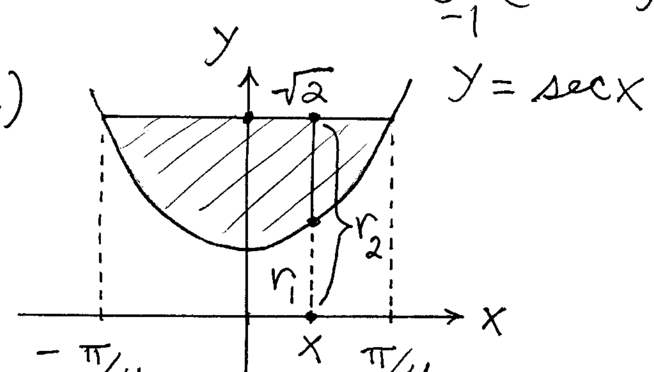
$x=2, \; x=-1$

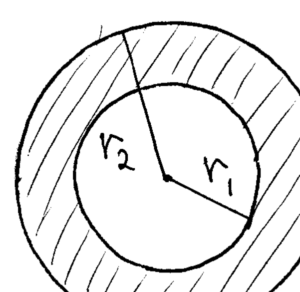
$$\text{Volume} = \pi \int_{-1}^{2} (r_2)^2\,dx - \pi \int_{-1}^{2} (r_1)^2\,dx$$



$$= \pi \int_{-1}^{2} (x+3)^2\,dx - \pi \int_{-1}^{2} (x^2+1)^2\,dx$$

43.)

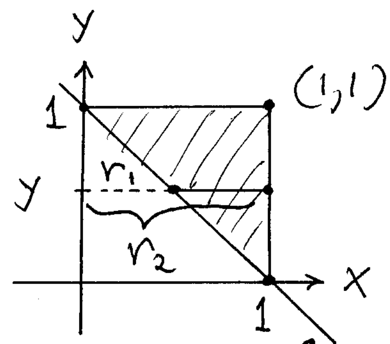




$$\text{Volume} = \pi \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (r_2)^2\,dx - \pi \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (r_1)^2\,dx$$

$$= \pi \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (\sqrt{2})^2 \, dx - \pi \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (\sec x)^2 \, dx$$

46.)

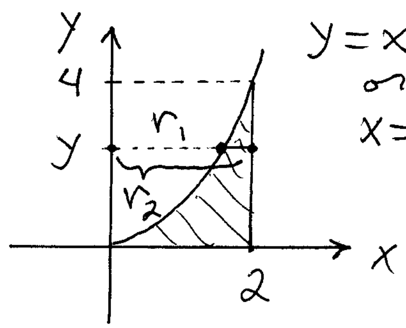


$y = 1 - x$ or
$x = 1 - y$

$$\text{Volume} = \pi \int_0^1 (r_2)^2 \, dy - \pi \int_0^1 (r_1)^2 \, dy$$

$$= \pi \int_0^1 (1)^2 \, dy - \pi \int_0^1 (1-y)^2 \, dy$$

47.)

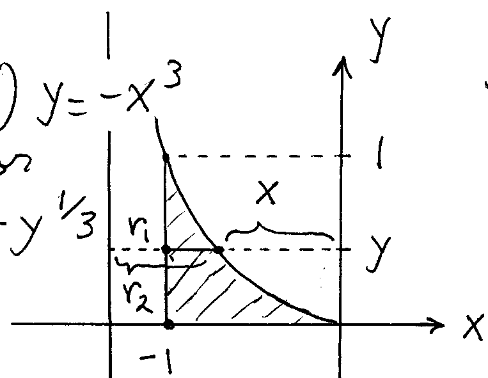


$y = x^2$
or
$x = \sqrt{y}$

$$\text{Volume} = \pi \int_0^4 (r_2)^2 \, dy - \pi \int_0^4 (r_1)^2 \, dy$$

$$= \pi \int_0^4 (2)^2 \, dy - \pi \int_0^4 (\sqrt{y})^2 \, dy$$

50.) $y = -x^3$
or
$x = -y^{1/3}$

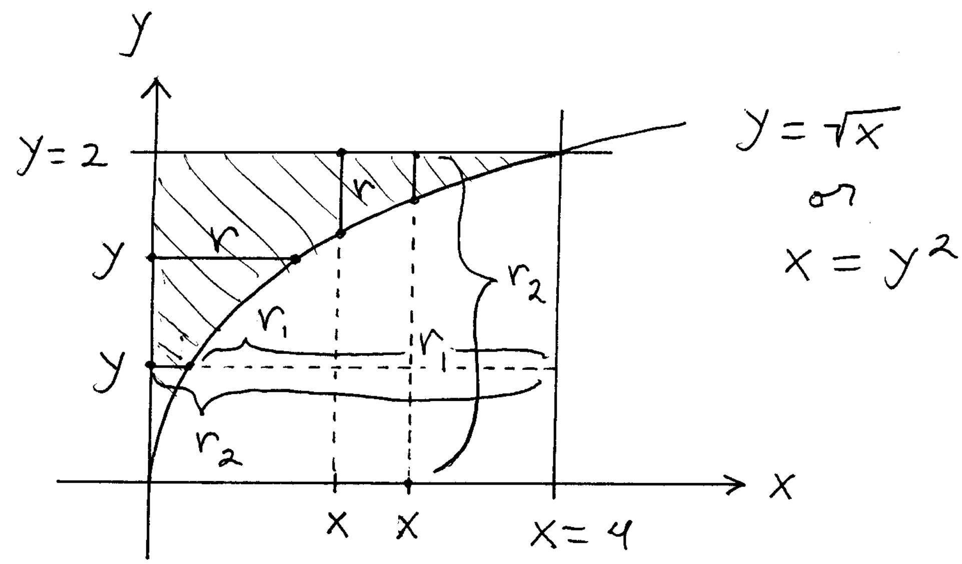


$x = -2$

$$\text{Volume} = \pi \int_0^1 (r_2)^2 \, dy - \pi \int_0^1 (r_1)^2 \, dy$$

$$= \pi \int_0^1 \left(-y^{1/3} - (-2)\right)^2 dy - \pi \int_0^1 (\ 1\ )^2 \, dy$$

51.)



a.) Volume $= \pi \int_0^4 (r_2)^2\,dx - \pi \int_0^4 (r_1)^2\,dx$

$= \pi \int_0^4 (2)^2\,dx - \pi \int_0^4 (\sqrt{x})^2\,dx$

b.) Volume $= \pi \int_0^2 (\text{radius})^2\,dy$

$= \pi \int_0^2 (y^2)^2\,dy$

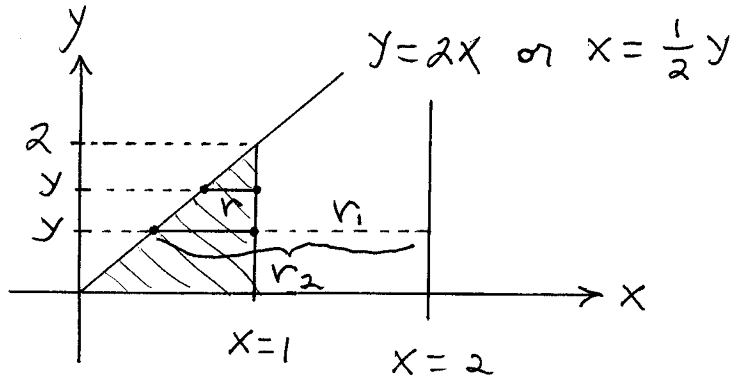
c.) Volume $= \pi \int_0^4 (\text{radius})^2\,dx$

$= \pi \int_0^4 (2-\sqrt{x})^2\,dx$

d.) Volume $= \pi \int_0^2 (r_2)^2\,dy - \pi \int_0^2 (r_1)^2\,dy$

$= \pi \int_0^2 (4)^2\,dy - \pi \int_0^2 (4-y^2)^2\,dy$

52.)



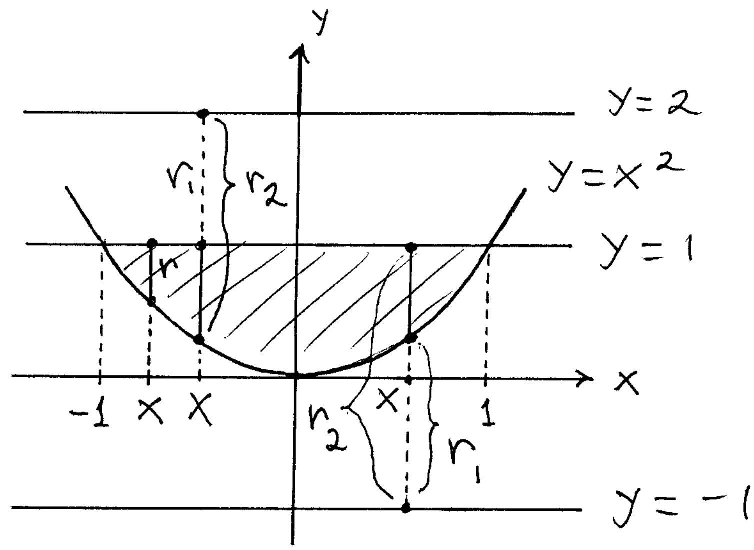
a.) Volume $= \pi \int_0^2 (\text{radius})^2\, dy$

$= \pi \int_0^2 (1-\frac{1}{2}y)^2\, dy$

b.) Volume $= \pi \int_0^2 (r_2)^2\, dy - \pi \int_0^2 (r_1)^2\, dy$

$= \pi \int_0^2 (2-\frac{1}{2}y)^2\, dy - \pi \int_0^2 (1)^2\, dy$

53.)



a.) Volume $= \pi \int_{-1}^1 (\text{radius})^2\, dx$

$= \pi \int_{-1}^1 (1-x^2)^2\, dx$

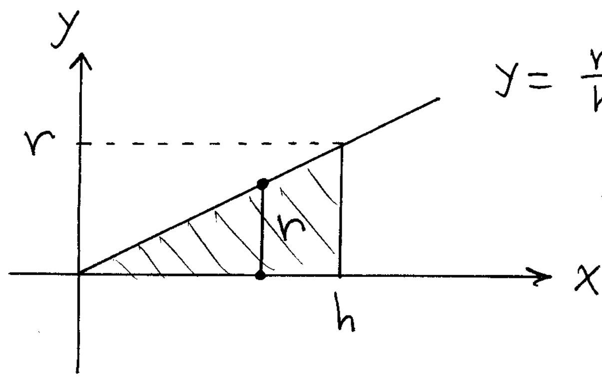
b.) Volume $= \pi \int_{-1}^1 (r_2)^2\, dx - \pi \int_{-1}^1 (r_1)^2\, dx$

$= \pi \int_{-1}^1 (2-x^2)^2\, dx - \pi \int_{-1}^1 (1)^2\, dx$

c.) Volume $= \pi \int_{-1}^{1} (r_2)^2 dx - \pi \int_{-1}^{1} (r_1)^2 dx$

$= \pi \int_{-1}^{1} (2)^2 dx - \pi \int_{-1}^{1} (x^2+1)^2 dx$

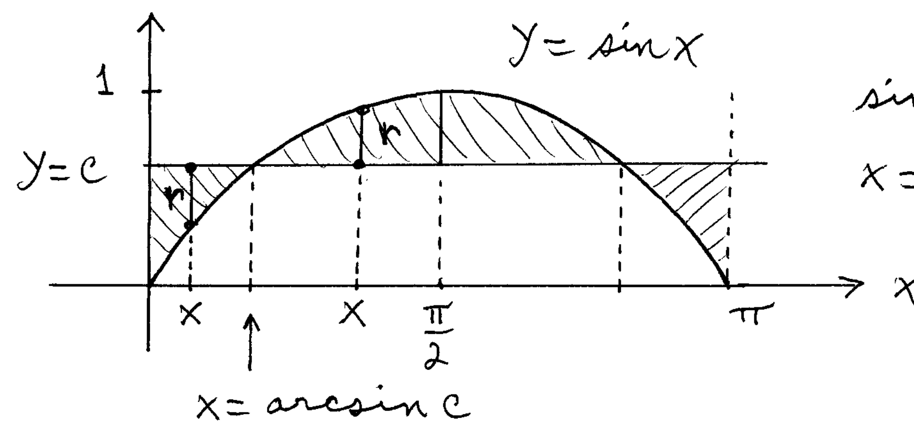
58.)



$y = \frac{r}{h} x$; rotate region about x-axis to form cylinder

$$\text{Volume} = \pi \int_0^h (\text{radius})^2 dx$$
$$= \pi \int_0^h \left(\frac{r}{h} x\right)^2 dx$$
$$= \pi \cdot \frac{r^2}{h^2} \int_0^h x^2 dx$$
$$= \pi \cdot \frac{r^2}{h^2} \cdot \frac{x^3}{3} \Big|_0^h$$
$$= \pi \cdot \frac{r^2}{h^2} \cdot \frac{h^3}{3}$$
$$= \frac{1}{3} \pi r^2 h$$

63.)



$\sin x = c \rightarrow$

$x = \arcsin c$

Use symmetry about $x = \frac{\pi}{2}$:

Volume

$$V(c) = 2 \int_0^{\arcsin c} (c - \sin x)^2 \, dx$$

$$+ \; 2 \int_{\arcsin x}^{\frac{\pi}{2}} (\sin x - c)^2 \, dx$$