

Math 21B

Kouba

Parametric Equations

## Parametric Equations

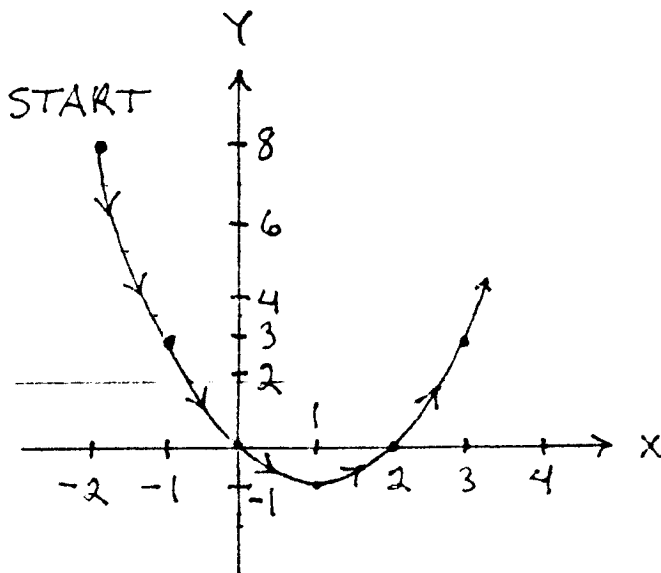
Ex: The position  $(x, y)$  of a particle at time  $t$  is given parametrically by

$$\begin{cases} x = t - 2 \\ y = t^2 - 6t + 8 \end{cases} \text{ for } t \geq 0.$$

a.) Find the particle's position when  $t = 0, 1, 2, 3, 4, 5$ :

$t$ :	0	1	2	3	4	5
$x$ :	-2	-1	0	1	2	3
$y$ :	8	3	0	-1	0	3

b.) Sketch the path and indicate the direction of motion:



c.) Remove parameter  $t$  and write the equation of the path in terms of  $x$  and  $y$  only:

$$\begin{cases} X = t - 2 \\ Y = t^2 - 6t + 8 \end{cases} \rightarrow t = X + 2 \rightarrow (\text{SUB}) \rightarrow$$

$$Y = (X + 2)^2 - 6(X + 2) + 8 \rightarrow$$

$$Y = X^2 + 4X + 4 - 6X - 12 + 8 \rightarrow$$

$$\boxed{Y = X^2 - 2X} \quad (\text{parabola})$$

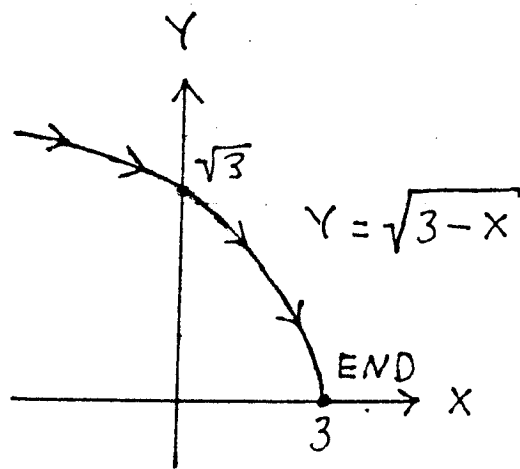
Ex: For each parametric representation write an equation of the path in terms of  $X$  and  $Y$  only; sketch the path in the  $XY$ -plane; indicate the direction of motion:

$$1.) \begin{cases} X = t + 1 \\ Y = \sqrt{2 - t} \end{cases} \text{ for } t \leq 2$$

$$\rightarrow t = X - 1 \rightarrow (\text{SUB}) \rightarrow$$

$$Y = \sqrt{2 - (X - 1)} = \sqrt{3 - X} \rightarrow$$

$$\boxed{Y = \sqrt{3 - X}}$$

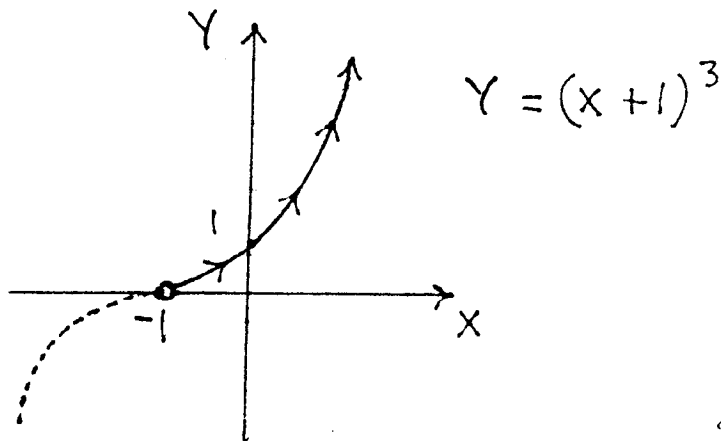


$$2.) \begin{cases} X = e^t - 1 \\ Y = e^{3t} \end{cases} \text{ for } -\infty < t < \infty \rightarrow \begin{cases} X = e^t - 1 \\ Y = (e^t)^3 \end{cases}$$

$$\rightarrow \begin{cases} e^t = X + 1 \\ Y = (e^t)^3 \end{cases}$$

$$\rightarrow \boxed{Y = (X + 1)^3}$$

(cubic)



$$3.) \begin{cases} x = 3 \sin t \\ y = 3 \cos t \end{cases} \text{ for } 0 \leq t \leq 2\pi \rightarrow$$

$$\begin{aligned} x^2 + y^2 &= (3 \sin t)^2 + (3 \cos t)^2 \\ &= 9 \sin^2 t + 9 \cos^2 t \\ &= 9 (\sin^2 t + \cos^2 t) \\ &= 9(1) \rightarrow \end{aligned}$$

$$\boxed{x^2 + y^2 = 9} \quad (\text{circle})$$

