Math 21B
Kouba
Estimating the Value of a Definite Integral

Suppose that the integral $\int_a^b f(x)\,dx$ is too difficult (or impossible) to compute, or that you are simply required to estimate its exact value. The following two methods offer possible ways to compute an estimate and measure its accuracy.

1.) TRAPEZOIDAL RULE

a.) Divide the interval $[a,b]$ into n equal parts, each of length $h = \dfrac{b-a}{n}$.

b.) Let $a = x_0, x_1, x_2, x_3, ..., x_{n-1}, x_n = b$ be the partition of the interval.

c.) The Trapezoidal Estimate for $\int_a^b f(x)\,dx$ is

$$T_n = \frac{h}{2}\,[f(x_0) + 2f(x_1) + 2f(x_2) + ... + 2f(x_{n-1}) + f(x_n)] \; .$$

d.) The Absolute Error is

$$|E_n| = \left| \int_a^b f(x)\,dx - T_n \right| \leq (b-a)\frac{h^2}{12}\left\{ \max_{a \leq x \leq b} |f''(x)| \right\} .$$

2.) SIMPSON'S RULE (NOTE: For this method n MUST be an even integer !)

a.) Divide the interval $[a,b]$ into n equal parts, each of length $h = \dfrac{b-a}{n}$.

b.) Let $a = x_0, x_1, x_2, x_3, ..., x_{n-1}, x_n = b$ be the partition of the interval.

c.) The Simpson Estimate for $\int_a^b f(x)\,dx$ is

$$S_n = \frac{h}{3}\,[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + ... + 4f(x_{n-3}) + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)] \; .$$

d.) The Absolute Error is

$$|E_n| = \left| \int_a^b f(x)\,dx - S_n \right| \leq (b-a)\frac{h^4}{180}\left\{ \max_{a \leq x \leq b} |f^{(4)}(x)| \right\} .$$