Math 21B

Kouba

Center of Mass $(\bar{x}, \bar{y})$ of a Flat Plate of Variable Density

Consider a flat region R bounded above by the graph of $y = f(x)$ and below by the graph of $y = g(x)$ for $a \le x \le b$,. Assume the density at point (x, y) is $\delta(x, y) = k(x)$, a function of x only (not y.). The formulas for the coordinates of the center of mass $(\bar{x}, \bar{y})$ of region R (for this specific assumption about the variable density being a function of x only) are
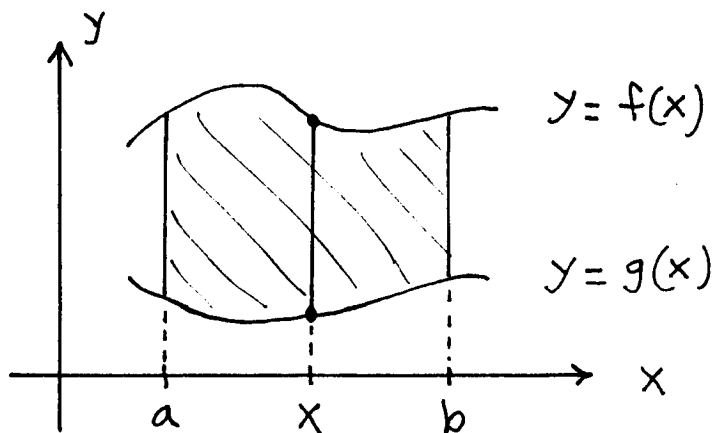
$$\bar{x} = \frac{\int_a^b x(f(x) - (g(x))\delta(x, y)\, dx}{\int_a^b (f(x) - g(x))\delta(x, y)\, dx} \quad \text{and} \quad \bar{y} = \frac{\int_a^b (1/2)((f(x))^2 - (g(x))^2)\delta(x, y)\, dx}{\int_a^b (f(x) - g(x))\delta(x, y)\, dx}$$

REMARK : The integral $\displaystyle\int_a^b (f(x) - g(x))\delta(x, y)\, dx$ represents the TOTAL MASS of the plate with variable density.

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