Math 21B (Winter 2006)
Kouba
Exam 3

Please PRINT your name here : KEY

Your Exam ID Number ____________

1. PLEASE DO NOT TURN THIS PAGE UNTIL TOLD TO DO SO.

2. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO, IN ANY WAY, ASSIST ANOTHER PERSON IN THE COMPLETION OF THIS EXAM. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO COPY ANSWERS FROM SOMEONE ELSE'S EXAM. PLEASE KEEP YOUR OWN WORK COVERED UP AS MUCH AS POSSIBLE DURING THE EXAM SO THAT OTHERS WILL NOT BE TEMPTED OR DISTRACTED. THANK YOU FOR YOUR COOPERATION.

3. No notes, books, or classmates may be used as resources for this exam. YOU MAY USE A CALCULATOR ON THIS EXAM.

4. Read directions to each problem carefully. Show all work for full credit. In most cases, a correct answer with no supporting work will NOT receive full credit. What you write down and how you write it are the most important means of your getting a good score on this exam. Neatness and organization are also important.

5. Make sure that you have 6 pages, including the cover page.

6. You will be graded on proper use of integral and derivative notation.

7. Do not use shortcuts when integrating using the method of integration by parts.

8. Include units on answers where units are appropriate.

9. You have until 11:50 a.m. to finish the exam.

1.) (10 pts.) In three (3) years a population of prairie dogs near Hotchkiss, Colorado, grew to 400. In five (5) years there were 1600 prairie dogs. Assuming exponential growth, what was the initial population of prairie dogs ?

assume $N = Ce^{kt}$;

$t = 3$ yrs, $N = 400 \to 400 = Ce^{3k}$

$t = 5$ yrs, $N = 1600 \to 1600 = Ce^{5k}$

$$\left.\begin{array}{l} 400 = Ce^{3k} \\ 1600 = Ce^{5k} \end{array}\right\} \to$$

$$C = \frac{400}{e^{3k}} \to 1600 = \frac{400}{e^{3k}} \cdot e^{5k} \to$$

$$4 = e^{2k} \to \ln 4 = 2k \to k = \frac{1}{2}\ln 4 = \ln 4^{\frac{1}{2}} = \ln 2$$

$\to$ initial amount

$$C = \frac{400}{e^{3\ln 2}} = \frac{400}{e^{\ln 2^3}} = \frac{400}{8} = 50 \text{ dogs}$$

2.) (10 pts.) Compute S_4, the Simpson's Rule estimate with $n = 4$, for the integral $\int_0^8 \sqrt{x^2+1}\,dx$.

0	2	4	6	8
x_0	x_1	x_2	x_3	x_4

$f(x) = \sqrt{x^2+1}$, $n = 4$, $h = \frac{8-0}{4} = 2$ so

$$S_4 = \frac{h}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)\right]$$

$$= \frac{2}{3}\left[\sqrt{1} + 4\sqrt{5} + 2\sqrt{17} + 4\cdot\sqrt{37} + \sqrt{65}\right]$$

$$\approx 33.72$$

3.) (10 pts. each) Use any method on the following indefinite integrals.

a.) $\int 3^{4x+7}\,dx$

(Let $u = 4x+7 \rightarrow du = 4\,dx \rightarrow \frac{1}{4}du = dx$)

$= \frac{1}{4}\int 3^u\,du = \frac{1}{4}\cdot\frac{1}{\ln 3}3^u + c$

$= \frac{1}{4\ln 3}\cdot 3^{4x+7} + c$

b.) $\int x^2 e^x\,dx$

(Let $u = x^2$, $dv = e^x dx$
$du = 2x\,dx$, $v = e^x$)

$= x^2e^x - 2\int xe^x\,dx$ (Let $u=x$, $dv = e^x dx$
$du = dx$, $v = e^x$)

$= x^2e^x - 2\left[xe^x - \int e^x dx\right]$

$= x^2e^x - 2xe^x + 2e^x + c$

c.) $\int \frac{x^2}{x^3-8}\,dx$

(Let $u = x^3-8 \rightarrow du = 3x^2dx$
$\rightarrow \frac{1}{3}du = x^2dx$)

$= \frac{1}{3}\int \frac{1}{u}\,du$

$= \frac{1}{3}\cdot\ln|u| + c$

$= \frac{1}{3}\ln|x^3-8| + c$

d.) $\int \sin^2(4x)\, dx = \int \frac{1}{2}(1 - \cos 2(4x))\, dx$

$= \frac{1}{2}\int (1 - \cos 8x)\, dx$

$= \frac{1}{2}\left(x - \frac{1}{8}\sin 8x\right) + c$

e.) $\int \sec^7 x \tan^3 x\, dx = \int \sec^6 x \cdot \tan^2 x \cdot \sec x \tan x\, dx$

$= \int \sec^6 x \cdot (\sec^2 x - 1) \cdot \sec x \tan x\, dx$

$= \int (\sec^8 x - \sec^6 x) \cdot \sec x \tan x\, dx$

(Let $u = \sec x \rightarrow du = \sec x \tan x\, dx$)

$= \int (u^8 - u^6)\, du = \frac{1}{9}u^9 - \frac{1}{7}u^7 + c$

$= \frac{1}{9}\sec^9 x - \frac{1}{7}\sec^7 x + c$

f.) $\int \frac{x^3}{x^2 - 4}\, dx$

$$\begin{array}{r} x \\ x^2 - 4 \;\big)\, \overline{x^3} \\ -(x^3 - 4x) \\ \hline 4x \end{array}$$

$= \int \left[x + \frac{4x}{x^2 - 4}\right] dx$

$= \frac{x^2}{2} + 2\ln|x^2 - 4| + c$

g.) $\displaystyle\int \frac{x^3}{\sqrt{4-x^2}}\,dx \qquad (\text{Let } x = 2\sin\theta \rightarrow dx = 2\cos\theta\, d\theta)$

$$= \int \frac{8\sin^3\theta}{\sqrt{4-4\sin^2\theta}} \cdot 2\cos\theta\, d\theta$$

$$= \frac{16}{2}\int \frac{\sin^3\theta \cdot \cos\theta}{\sqrt{\cos^2\theta}}\, d\theta = 8\int \frac{\sin^3\theta \cdot \cancel{\cos\theta}}{\cancel{\cos\theta}}\, d\theta$$

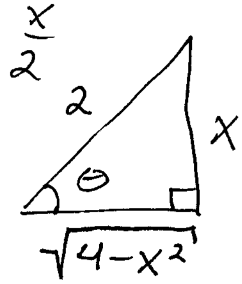
$$= 8\int \sin\theta\,(\sin^2\theta)\, d\theta = 8\int \sin\theta\,(1-\cos^2\theta)\, d\theta$$

$$= 8\int (\sin\theta - \cos^2\theta \sin\theta)\, d\theta$$

$$= 8\left(-\cos\theta + \frac{1}{3}\cos^3\theta\right) + C$$

$$= -8 \cdot \frac{\sqrt{4-x^2}}{2} + \frac{8}{3}\left(\frac{\sqrt{4-x^2}}{2}\right)^3 + C$$

$\sin\theta = \frac{x}{2}$



h.) $\displaystyle\int \frac{1}{\cos x \cdot (\sin^2 x + 1)}\,dx$ **(HINT : First multiply and divide by $\cos x$.)**

$$= \int \frac{\cos x}{\cos^2 x \cdot (\sin^2 x + 1)}\, dx = \int \frac{\cos x}{(1-\sin^2 x)(\sin^2 x + 1)}\, dx$$

$(\text{Let } u = \sin x \rightarrow du = \cos x\, dx)$

$$= \int \frac{1}{(1-u^2)(u^2+1)}\, du = \int \frac{1}{(1-u)(1+u)(u^2+1)}\, du$$

$$= \int \left[\frac{A}{1-u} + \frac{B}{1+u} + \frac{Cu+D}{u^2+1}\right] du$$

$\{ A(1+u)(u^2+1) + B(1-u)(u^2+1) + (Cu+D)(1-u)(1+u) = 1$

Let $u = 1$: $4A = 1 \rightarrow A = 1/4$

Let $u = -1$: $4B = 1 \rightarrow B = 1/4$

Let $u = i$: $(Ci + D)(1 - i^2) = 1 \rightarrow$

$(2C)i + (2D) = (0)i + (1) \rightarrow C = 0,\ D = 1/2 \}$

$$= \int \left[\frac{1/4}{1-u} + \frac{1/4}{1+u} + \frac{1/2}{u^2+1}\right] du$$

$$= -\frac{1}{4}\ln|1-u| + \frac{1}{4}\ln|1+u| + \frac{1}{2}\arctan u + C$$

$$= -\frac{1}{4}\ln|1-\sin x| + \frac{1}{4}\ln|1+\sin x| + \frac{1}{2}\arctan(\sin x) + C.$$

The following EXTRA CREDIT problem is OPTIONAL. It is worth 10 points.

1.) Integrate : $\int \frac{x^3 e^{x^2}}{(x^2+1)^2}\,dx \;=\; \int \frac{x}{(x^2+1)^2}\cdot x^2 e^{x^2}\,dx$

$$\left(\text{Let } u = x^2 e^{x^2}, \quad dv = \frac{x}{(x^2+1)^2}\,dx \right.$$

$$du = (x^2\cdot 2xe^{x^2} + 2xe^{x^2})\,dx$$

$$= 2x(x^2+1)e^{x^2}\,dx,$$

$$\left. V = \frac{-1}{2}\cdot\frac{1}{x^2+1} \right)$$

$$= \frac{-1}{2}\cdot\frac{x^2}{x^2+1}\cdot e^{x^2} + \int xe^{x^2}\,dx$$

$$= \frac{-1}{2}\cdot\frac{x^2}{x^2+1}\cdot e^{x^2} + \frac{1}{2}e^{x^2} + C$$