Math 21C
Kouba
Discussion Sheet 4

1.) Find three positive numbers whose sum is 27 and such that the sum of their squares is as small as possible.

2.) Determine the dimensions of an open rectangular box having volume 32 ft.^{3} and requiring the least amount of material for its construction.

3.) A closed rectangular box with a volume of 16 ft.^{3} is made from two kinds of material. The top and bottom are made of material costing 10 cents per square foot and the sides from material costing 5 cents per square foot. Compute the dimensions of the least expensive box.

4.) Find all points on the surface $x^2 - yz = 5$ that are closest to the origin.

5.) Find the point (x, y) which minimizes the sum of the *squares* of the distances from (x, y) to the points $(0, 0), (0, 3)$, and $(6, 0)$.

6.) Let R be the region bounded by the graphs of $y = x^2$ and $y = 3x$.
   a.) Describe R using vertical cross-sections.
   b.) Describe R using horizontal cross-sections.

7.) Let R be the region inside the circle of radius 5 centered at $(3, 4)$ and to the right of the line $x = 3$.
   a.) Describe R using vertical cross-sections.
   b.) Describe R using horizontal cross-sections.

8.) Let R be the triangular region with vertices $(0, 0), (2, 0)$, and $(3, 2)$.
   a.) Describe R using vertical cross-sections.
   b.) Describe R using horizontal cross-sections.

9.) Sketch each of the following regions described in two-dimensional space.
   a.) $0 \le x \le 3$, $2 \le y \le 4$ b.) $0 \le x \le 3$, $\sqrt{x} \le y \le x + 1$
   c.) $1 \le x \le 3$, $0 \le y \le \ln x$ d.) $0 \le y \le \ln 3$, $e^y \le x \le 3$
   e.) $0 \le y \le 1$, $0 \le x \le \arcsin y$

10.) Evaluate the following double integrals.

a.) $\displaystyle\int_0^1 \int_{x^2}^{x} xy^2 \, dy \, dx$ b.) $\displaystyle\int_{\pi/2}^{\pi} \int_0^{x^2} (1/x) \cos(y/x) \, dy \, dx$

(Beware of the next two.)

c.) $\int_0^1 \int_{4x}^4 e^{-y^2}\, dy\, dx$ d.) $\int_0^2 \int_{y/2}^1 \cos(x^2)\, dx\, dy$

11.) Use a double integral to find the area of the region in problem 6.)

12.) Consider the tetrahedron with vertices $(0,0,0), (4,0,0), (0,3,0)$, and $(0,0,2)$.

a.) It's top surface is a plane. Find an equation for this plane.

b.) Set up but do not evaluate a double integral which represents the volume of the tetrahedron.

13.) Consider a flat plate lying in the region bounded by the graphs of $y = e^x, x = 0$, and $y = 2$. Assume that density at point (x, y) is given by $\delta(x, y) = x^2y^3 + 1$.

a.) Set up but do not evaluate a double integral which represents the area of the plate.

b.) Set up but do not evaluate a double integral which represents the mass of the plate.

c.) Set up but do not evaluate double integrals which represent the centroid of the plate.

d.) Set up but do not evaluate double integrals which represent the center of mass of the plate.

e.) Set up but do not evaluate double integrals which represent the moment of inertia of the plate about

i.) the origin.

ii.) the x-axis.

iii.) the line $x = 4$.

THE FOLLOWING PROBLEM IS FOR RECREATIONAL PURPOSES ONLY.

14.) A snail is at the bottom of a well which is 100 feet deep. Each day the snail climbs up 6 feet and down 4 feet. In how many days will the snail reach the top of the well ?