Math 21C
Kouba
More Sequences and Series

1.) Consider the sequence given by $a_1 = 3$ and $a_{n+1} = \frac{n}{n+1} a_n$ for $n = 1, 2, 3, \ldots$. Does the series $\sum_{n=1}^{\infty} (-1)^n a_n$ converge or diverge ?

2.) Consider the sequence given by $a_1 = 3, a_2 = 1$, and $a_{n+2} = \frac{3a_{n+1}}{a_n + 1}$ for $n = 1, 2, 3, \ldots$. Does the series $\sum_{n=1}^{\infty} \frac{1}{(a_n)^n}$ converge or diverge ?

3.) a.) For what values of n, if any, is $n^2 < (\ln n)^{\ln n}$?

b.) Does the series $\sum_{n=2}^{\infty} \frac{1}{(\ln n)^{\ln n}}$ converge or diverge ?

4.) Let $s_n = \sum_{k=1}^{n} \ln \left(\frac{k}{k+1} \right)$. Evaluate $\lim_{n \to \infty} (\ln(3n) + s_n)$.

5.) Consider the sequence given by $a_n = \frac{e^n n!}{n^n}$ for $n = 1, 2, 3, \ldots$.

a.) Show that $a_{n+1} > a_n$ for all values of n.

b.) What, if anything, can be said about $\lim_{n \to \infty} a_n$?

6.) Use any test to determine the convergence or divergence of each series.

a.) $\sum_{n=1}^{\infty} \frac{1}{n^n}$ b.) $\sum_{n=1}^{\infty} \frac{1}{n^{1/n}}$ c.) $\sum_{n=2}^{\infty} \frac{1}{n^{(1+1/n)}}$ d.) $\sum_{n=1}^{\infty} n^{-1} \sin \left[\frac{(2n-1)\pi}{2} \right]$

e.) $\sum_{n=1}^{\infty} \frac{\ln n}{n}$ f.) $\sum_{n=1}^{\infty} \frac{\ln n}{n^{1.2}}$ g.) $\sum_{n=1}^{\infty} \frac{(2.71)^n n!}{n^n}$ h.) $\sum_{n=1}^{\infty} \frac{e^n n!}{n^n}$

i.) $\sum_{n=4}^{\infty} \frac{1}{n(\ln n)^4}$ j.) $\sum_{n=3}^{\infty} \frac{(\ln n)^4}{n}$ k.) $\sum_{n=2}^{\infty} \frac{(\sin(1/n))^4}{n}$