Math 21C DHC
Kouba
Discussion Sheet 3

1.) Assume that $z = f(x, y)$, $x = ut$, and $y = 3u + 2t$. Determine $\dfrac{\partial z}{\partial u}$, $\dfrac{\partial z}{\partial t}$, and $\dfrac{\partial^2 z}{\partial u^2}$.

2.) Assume that $w = g(xy)$, $x = u^2 e^t$, and $y = \sin(u - t)$. Determine $\dfrac{\partial w}{\partial u}$, $\dfrac{\partial w}{\partial t}$, and $\dfrac{\partial^2 w}{\partial t^2}$.

3.) Find and classify critical points for $f(x, y) = 6x^2 y - 12y^2 - x^4 + 18y$.

4.) Determine the absolute minimum and maximum values for
$f(x, y) = 3x^2 + y^4$ defined on and inside the square with corners $(-1, 1)$, $(-1, -2)$, $(2, 1)$, and $(2, -2)$.

5.) Build an open (no top) rectangular box with volume 4 cubic feet. What dimensions (length, width, and height) will result in a box of minimum surface area ? (You need not verify that the critical point determines a minimum.)

6.) Find three numbers whose sum is 9 and so that the sum of the squares of two of the numbers and four times the square of the third number is a minimum. (You need not verify that the critical point determines a minimum.)

7.) Evaluate the following double integrals.

a.) $\displaystyle\int_0^2 \int_0^1 4xye^{x^2+y^2}\, dy\, dx$ b.) $\displaystyle\int_0^1 \int_{2x}^2 e^{y^2}\, dy\, dx$ c.) $\displaystyle\int_0^4 \int_{\sqrt{x}}^2 \frac{2x}{1+y^5}\, dy\, dx$

8.) Consider the tetrahedron with vertices $(0, 0, 0)$, $(3, 0, 0)$, $(0, 4, 0)$, and $(0, 0, 5)$.

a.) Determine an equation for the top surface (plane) of the solid.
b.) SET UP but DO NOT EVALUATE a double integral which represents the volume of the solid.

9.) Consider a flat plate of variable density lying in the region bounded by the graphs of $y = x$, $y = 2x$, and $y = 6$. Assume that the density (kg./cm.^{2}) at the point $P = (x, y)$ is numerically equal to 3 plus the distance from P to the origin. SET UP but DO NOT EVALUATE a double integral which represents the total mass of the plate.