Math 21C DHC
Kouba
Discussion Sheet 4

1.) Consider the tetrahedron R with vertices $(2,0,3)$, $(0,0,3)$, $(0,4,0)$, and $(0,4,3)$.

a.) Sketch R in three dimensional space.
b.) Describe R using rectangular coordinates by first projecting R onto
i.) the xy-plane.
ii.) the yz-plane.
iii.) the xz-plane.
c.) SET UP but DO NOT EVALUATE a double integral which represents the volume of R.
d.) SET UP but DO NOT EVALUATE a triple integral which represents the volume of R.

2.) Evaluate $\displaystyle\int_0^{\pi/2}\int_0^{z}\int_0^{y}\cos(x+y+z)\,dx\,dy\,dz$.

3.) Evaluate the following integrals by first converting to polar (cylindrical) coordinates.

a.) $\displaystyle\int_{1/\sqrt{2}}^{1}\int_{\sqrt{1-x^2}}^{x}\frac{1}{\sqrt{x^2+y^2}}\,dy\,dx$ b.) $\displaystyle\int_0^{3/2}\int_{\sqrt{3}x}^{\sqrt{9-x^2}}\int_0^{2}\sqrt{x^2+y^2}\,dz\,dy\,dx$

4.) Sketch the solid in three dimensional space whose volume is given by the following integral.

a.) $\displaystyle\int_{\pi/4}^{\pi/2}\int_0^{\csc\theta}(5r-r^2\cos\theta-r^2\sin\theta)\,dr\,d\theta$ b.) $\displaystyle\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{x^2+y^2}^{\sqrt{2-x^2-y^2}}1\,dz\,dy\,dx$

5.) Consider the rectangular box in three dimensional space bounded by the planes $x=0$, $x=4$, $y=0$, $y=3$, $z=0$, and $z=5$. Assume that the density at the point $P=(x,y,z)$ is numerically equal to 7 plus the distance from P to the point $(3,4,5)$. SET UP but DO NOT EVALUATE a triple integral which represents the moment of inertia of this solid about

a.) the origin. b.) the z-axis.

6.) The center of mass of a solid R of mass M is located at $(0,0,0)$. Its moment of inertia about the x-axis is I.

a.) Find the moment of inertia for R about a line parallel to the x-axis and k units from the x-axis.
b.) About which such line parallel to the x-axis is the moment of inertia the least ?