

Math 21C DHC
Kouba
Discussion Sheet 5

1.) Determine the limits of the following sequences.

a.) $\left\{ (1 + 3/n)^n \right\}$ b.) $\left\{ \frac{3n + 1}{5n - 4} \right\}$ c.) $\left\{ \frac{100,000^n}{n!} \right\}$ d.) $\left\{ 2 + (-1)^n \right\}$

e.) $\left\{ \frac{2 + (-1)^n}{n} \right\}$ f.) $\left\{ \sin(n\pi) \right\}$ g.) $\left\{ \cos(n\pi) \right\}$ h.) $\left\{ \cos(\pi/n) \right\}$

i.) $\left\{ n \sin(\pi/n) \right\}$ j.) $\left\{ \sum_{i=1}^n \frac{1}{i(i+1)} \right\}$ k.) $\left\{ \sum_{i=1}^n \left(3 + \frac{i}{n} \right)^2 \left(\frac{1}{n} \right) \right\}$

2.) Convert, but DO NOT EVALUATE, the triple integral $\int_{\pi/6}^{\pi/2} \int_0^{\pi/4} \int_0^{\sec \phi} \rho^3 \sin \phi \cos \phi \, d\rho \, d\phi \, d\theta$ to

- a.) rectangular coordinates.
- b.) cylindrical coordinates.

3.) Convert, but DO NOT EVALUATE, the triple integral $\int_0^{\pi} \int_0^{2 \sin \theta} \int_0^{r^2} r^3 \sin \theta \cos \theta \, dz \, dr \, d\theta$ to

- a.) rectangular coordinates.
- b.) spherical coordinates.

4.) Define the function $f(t) = \int_t^1 e^{x^2} \, dx$. Determine the average value of f over the interval $[0, 1]$.

5.) The following equations are given in spherical coordinates. Sketch their graphs in three dimensional space.

- a.) $\rho = 1$ b.) $\rho = \sec \phi$ c.) $\rho = \cos \phi$
- d.) $\rho = \csc \phi$ e.) $\rho = \csc \phi \sec \theta$ f.) $\rho = \sec \theta$

6.) Define a sequence in the following way. Let $a_1 = 2$ and let $a_{n+1} = \frac{72}{1 + a_n}$ for $n = 1, 2, 3, 4, \dots$

- a.) Determine a_2 , a_3 , and a_4 .
- b.) Determine the limit of the sequence.