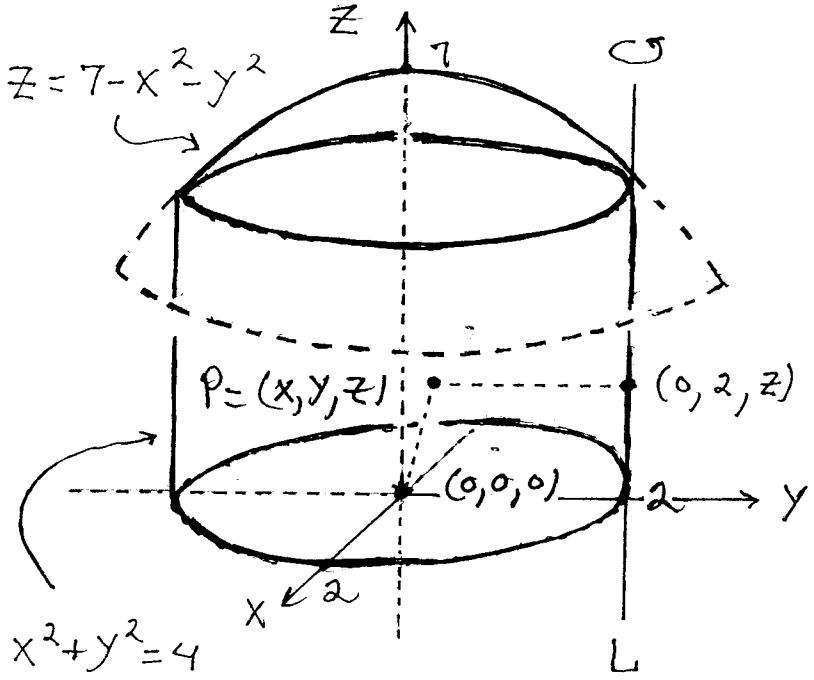
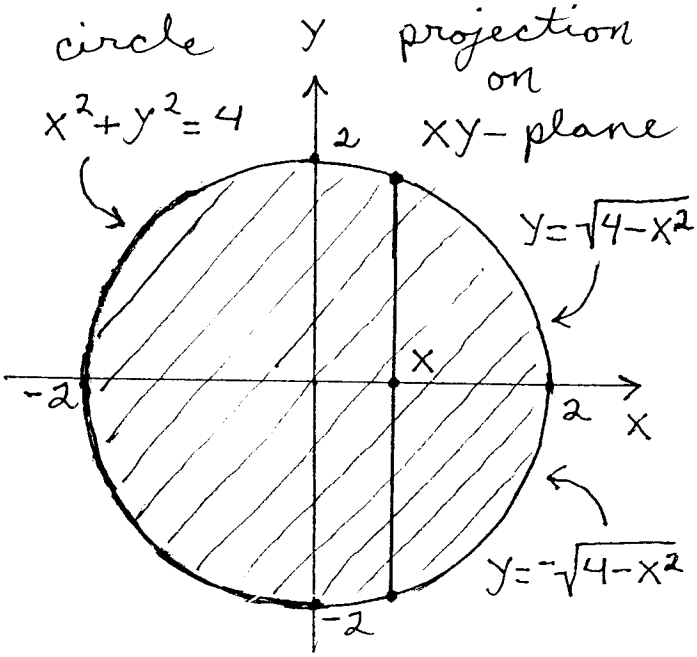
Math 21C
Kouba
Triple Integral in Rectangular Coordinates, an Example

Consider the solid region R above the plane $z = 0$, inside the cylinder $x^2 + y^2 = 4$, and below the paraboloid $z = 7 - x^2 - y^2$. Assume that the density at point $P = (x, y, z)$ is numerically equal to the distance from P to the origin. SET UP but do not evaluate a triple integral in rectangular coordinates, which represents the *moment of inertia* of R about a line parallel to the z-axis and passing through the edge of the solid.





$$R: \begin{cases} -2 \le x \le 2 \\ -\sqrt{4-x^2} \le y \le \sqrt{4-x^2} \\ 0 \le z \le 7 - x^2 - y^2 \end{cases};$$

density at P is $\delta(x, y, z) = \sqrt{x^2 + y^2 + z^2}$;

distance from P to line L is

$$\text{distance} = \sqrt{(x-0)^2 + (y-2)^2 + (z-z)^2}$$

$$= \sqrt{x^2 + (y-2)^2} \text{ ; then}$$

$$\text{M. of I.} = \int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{0}^{7-x^2-y^2} \left(\sqrt{x^2 + (y-2)^2}\right)^2 \cdot \sqrt{x^2 + y^2 + z^2} \, dz \, dy \, dx$$