Math 21C
Kouba
Discussion Sheet 2

1.) Determine convergence or divergence of each series using the test indicated. I suggest that you read all of the assumptions and conclusions for each test as you do each problem.

a.) $\displaystyle\sum_{n=3}^{\infty} \frac{2n+3}{3n+2}$ (Use the nth term test.)

b.) $\displaystyle\sum_{n=4}^{\infty} 7\frac{(-2)^{n+1}}{3^{n-1}}$ (Use the geometric series test.)

c.) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{\sqrt{2}}}$ (Use the p-series test.)

d.) $\displaystyle\sum_{n=2}^{\infty} \frac{n}{n^2+4}$ (Use the integral test.)

e.) $\displaystyle\sum_{n=1}^{\infty} \left\{\frac{1}{\sqrt{n+1}} - \frac{1}{\sqrt{n+2}}\right\}$ (Use the sequence of partial sums test.)

f.) $\displaystyle\sum_{n=2}^{\infty} \frac{n-1}{n^3+2}$ (Use the comparison test.)

g.) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3+7n^2-3}{n^4-4n+9}$ (Use the limit comparison test.)

h.) $\displaystyle\sum_{n=0}^{\infty} \frac{3^{n-1}}{(n+1)!}$ (Use the ratio test.)

i.) $\displaystyle\sum_{n=1}^{\infty} \left(1.01 - \frac{5}{n^3}\right)^n$ (Use the root test.)

2.) Use any test to determine the convergence or divergence of each series.

a.) $\displaystyle\sum_{n=1}^{\infty} \cos(1/n^2)$ b.) $\displaystyle\sum_{n=1}^{\infty} \sin(1/n^2)$ c.) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ d.) $\displaystyle\sum_{n=1}^{\infty} 3\left(2^{-n}\right)$

e.) $\displaystyle\sum_{n=1}^{\infty} 2\left(n^{-3}\right)$ f.) $\displaystyle\sum_{n=0}^{\infty} \sqrt{\frac{n+3}{n^3+8}}$ g.) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n+3^n}{5^n-2^n}$ h.) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{1+2+3+\cdots+n}$

i.) $\displaystyle\sum_{n=3}^{\infty} \frac{1}{\ln n}$ j.) $\displaystyle\sum_{n=2}^{\infty} \frac{\ln n}{n^3}$ k.) $\displaystyle\sum_{n=3}^{\infty} \frac{1}{\left(n+1/n\right)^n}$ l.) $\displaystyle\sum_{n=2}^{\infty} \frac{(2n)!}{n^2+100}$

3.) Consider the series $\displaystyle\sum_{n=3}^{\infty} \frac{1}{4n^2-1}$.

a.) Use the limit comparison test to show that the series converges.

b.) Use partial fractions then the sequence of partial sums to find the exact value of this series.

4.) Find the exact value of the following convergent series :

$$\frac{3^{-3}}{10^1} - \frac{3^{-1}}{10^2} + \frac{3^1}{10^3} - \frac{3^3}{10^4} + \frac{3^5}{10^5} - \frac{3^7}{10^6} + \cdots$$

5.) Use a geometric series to convert the decimal number 0.2525252525... to a fraction.

6.) The series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges.

a.) Use equation (*) to determine between which two numbers the partial sum $S_{50} = \sum_{i=1}^{50} \frac{1}{i}$ lies.

b.) What should n be in order that the partial sum $S_n = \sum_{i=1}^{n} \frac{1}{i}$ be at least 20 ?

c.) What is the largest value of n for which the partial sum $S_n = \sum_{i=1}^{n} \frac{1}{i}$ does not exceed 50 ?

7.) The series $\sum_{n=1}^{\infty} \frac{1}{n^3}$ converges.

a.) Compute the partial sum $S_5 = \sum_{i=1}^{5} \frac{1}{i^3}$. Use (*)(*) to estimate the resulting error.

b.) What should n be in order that the partial sum $S_n = \sum_{i=1}^{n} \frac{1}{i^3}$ estimate the exact value of the series with error at most 0.0001 ?

"What is important is to keep learning, to enjoy challenge, and to tolerate ambiguity. In the end there are no certain answers." – Martina Horner