Math 21C
Kouba
Discussion Sheet 3

1.) Determine convergence or divergence of each series using the test indicated.

a.) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{7}{n^2+3}$ (Use the alternating series test.)

b.) $1+\frac{1}{3}-\frac{1}{3^2}+\frac{1}{3^3}+\frac{1}{3^4}-\frac{1}{3^5}+\frac{1}{3^6}+\frac{1}{3^7}-\frac{1}{3^8}+\ldots$ (Use the absolute convergence test.)

2.) Determine if each of the following series, which contain both positive and negative terms, converges conditionally, converges absolutely, or diverges. Remember what the absolute convergence test says.

a.) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n+2}$ b.) $\displaystyle\sum_{n=4}^{\infty}(-1)^{n+1}\frac{1}{n^2+4}$ c.) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n+1}{n^2+99}$ (Use a derivative to show that $f(x)=\dfrac{x+1}{x^2+99}$ decreases after some point.)

d.) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n^2+50}{n^5+50}$ e.) $\frac{5}{1^4}+\frac{5}{2^4}-\frac{5}{3^4}-\frac{5}{4^4}+\frac{5}{5^4}+\frac{5}{6^4}-\frac{5}{7^4}-\frac{5}{8^4}+\cdots$

f.) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n}\frac{(\ln n)^3}{n}$ (Use a derivative to show that $f(x)=\dfrac{(\ln x)^3}{x}$ decreases after some point.)

3.) Use the absolute ratio test to determine the interval of convergence for each series.

a.) $\displaystyle\sum_{n=1}^{\infty}\frac{x^n}{n+1}$ b.) $\displaystyle\sum_{n=1}^{\infty}\frac{x^n}{n^2+1}$ c.) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n}\frac{(x-1)^{n+1}}{n2^n}$ d.) $\displaystyle\sum_{n=1}^{\infty}\frac{x^{n-1}}{(n+1)!}$

e.) $\displaystyle\sum_{n=1}^{\infty}(n!)^2x^n$ f.) $\displaystyle\sum_{n=0}^{\infty}\frac{(n!)x^n}{n^{2n}}$ g.) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{n+1}{n+3}\right)^n x^n$ (HINT: Use the absolute root test.)

The following three problems illuminate the odd behavior of some infinite series, which contain both positive and negative terms. At the very least we are made aware of the need to use precise testing methods (e.g., alternating series test, absolute convergence test, sequence of partial sums test, geometric series test, etc.), and that our intuition sometimes may fail when applied to problems like these.

4.) a.) Consider the series $\displaystyle\sum_{n=1}^{\infty}(\cos(2n\pi)+\cos((2n+1)\pi))$.

i.) Write out the first 4 terms of the series.

ii.) Use a particular test to determine if the series converges or diverges.

b.) Consider the series $\sum_{n=1}^{\infty}(-1)^{n+1}$.

i.) Write out the first 8 terms of the series.

ii.) Use a particular test to determine if the series converges or diverges.

c.) The series in a.) and b.) appear to be the same, but the answers in a.) ii.) and b.) ii.) show otherwise.

5.) a.) Consider the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{n+1}$.

i.) Write out the first 8 terms of the series.

ii.) Use a particular test to determine if the series converges or diverges.

b.) Consider the series $\sum_{n=1}^{\infty}\left(\frac{2n-1}{2n}-\frac{2n}{2n+1}\right)$.

i.) Write out the first 4 terms of the series.

ii.) Use a particular test to determine if the series converges or diverges.

c.) The series in a.) and b.) appear to be the same, but the answers in a.) ii.) and b.) ii.) show otherwise.

6.) a.) Consider the series $\sum_{n=0}^{\infty}(-1/2)^{n}$.

i.) Write out the first 8 terms of the series.

ii.) Use a particular test to determine if the series converges or diverges.

b.) Consider the series $\sum_{n=0}^{\infty}\left(\frac{1}{2^{2n}}-\frac{1}{2^{2n+1}}\right)$.

i.) Write out the first 4 terms of the series.

ii.) Use a particular test to determine if the series converges or diverges.

c.) Evaluate each series and then their sum :

$$\left(1+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\cdots\right)+\left(-\frac{1}{2}-\frac{1}{8}-\frac{1}{32}-\frac{1}{128}-\cdots\right).$$

d.) The series in a.), b.), and c.) appear to be the same, and the answers in a.) ii.), b.) ii.), and c.) bear out that they are. What characteristic do the series in 6.) have that those in 4.) and 5.) do not ? The series (All are the same.) in 6.) is an *absolutely convergent* series, which converges to the same number no matter how the terms in the series are grouped or in what order they are added. This is not always true of *conditionally convergent* or *divergent* series.

"Nothing in life is to be feared. It is only to be understood." – Marie Curie