Math 21C
Kouba
Discussion Sheet 5

1.) Write each power series as an ordinary function.

a.) $\sum_{n=5}^{\infty} x^n$ b.) $\sum_{n=0}^{\infty} 2^n x^n$ c.) $\sum_{n=0}^{\infty} \frac{(-3)^{n+1} x^n}{5^{n-1}}$

d.) $\sum_{n=4}^{\infty} n x^{n-1}$ e.) $\sum_{n=0}^{\infty} n^2 x^{n-1}$ f.) $\sum_{n=1}^{\infty} \frac{x^{n+3}}{n}$

g.) $\sum_{n=1}^{\infty} (-1)^n \frac{x^n}{2^n n!}$ h.) $\sum_{n=2}^{\infty} (-1)^n \left(\frac{2}{5}\right)^{2n} \frac{x^{2n+1}}{2n+1}$

2.) Use any method to find the exact value of each of the following convergent series.

a.) $\sum_{n=0}^{\infty} 3\left(\frac{-2}{3}\right)^n$ b.) $\sum_{n=4}^{\infty} \frac{(-1)^{n+2}}{2^{n-3}}$ c.) $\sum_{n=1}^{\infty} n^2 \left(\frac{1}{2}\right)^n$

d.) $\sum_{n=0}^{\infty} n(n-1)\left(\frac{3}{4}\right)^{n+1}$ e.) $\sum_{n=0}^{\infty} \frac{(\ln 2)^n}{n!}$ f.) $\sum_{n=2}^{\infty} (-1)^n \frac{9^n}{(2n)!}$

3.) Find the distance between the points $(3, -2, 4)$ and $(2, -6, -4)$.

4.) Find an equation of the sphere whose diameter has endpoints $(2, 4, -5)$ and $(0, -2, 4)$.

5.) Find the center and radius of the following sphere: $x^2 + y^2 + z^2 = 2x - 4y + 6z - 5$

6.) Determine a formula (and sketch the surface) for the set of all points (x, y, z) in three-dimensional space which are

a.) 4 units from the point $(2, -3, 0)$.
b.) 3 units from the z-axis.
c.) 1/2 unit from the x-axis.
d.) 2 units from the plane $y = 3$.
e.) equidistant from the points $(3, 0, 0)$ and $(0, 0, 3)$.
f.) equidistant from the planes $z = 2$ and $z = 6$.
g.) equidistant from the planes $x = 3$ and $y = 2$.
h.) equidistant from the point $(0, 0, 2)$ and the xy-plane.

7.) a.) If vector $\vec{A} = \overrightarrow{(1, 0, -2)}$, then what is the unit vector in the same direction as $\vec{A}$?

b.) If vector $\vec{A} = \overrightarrow{(a, b, c)}$, and a, b, and c are not all zero, then what is the unit vector in the same direction as $\vec{A}$?

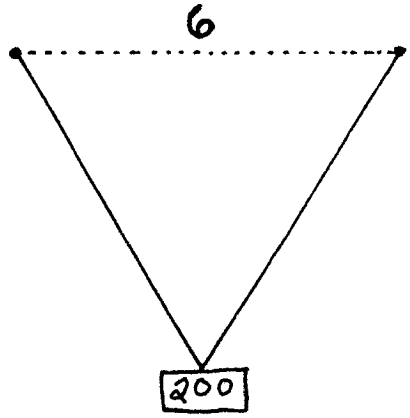
8.) Determine the vector $\vec{B}$, which starts at point $(1, -1, 0)$ and ends at point $(-1, 2, 6)$. Find a vector of length 2 pointing in the opposite direction of $\vec{B}$.

9.) Find two vectors of length 3 which are both perpendicular to

a.) vector $\overrightarrow{W} = 3\vec{i} + 4\vec{j}$.
b.) vector $\overrightarrow{W} = \vec{i} - 2\vec{j} + 2\vec{k}$.

10.) A sailboat starts at the origin $(0, 0)$, then sails
a.) 3 km NW, then turns and sails
b.) 2 km 60^o North of East, then turns and sails
c.) 4 km SE, then turns and sails
d.) 10 km 30^o South of West, and stops. What are the sailboat's coordinates now ?

11.) Two strong wires of equal length are hung from two supports which are at the same height and 6 feet apart. Each wire is attached to the same point of a 200 pound weight. What is the force of tension (in pounds) on each wire if the wires are each



a.) 5 feet long ?
b.) 20 feet long ?
c.) 3.1 feet long ?
d.) 3.01 feet long ?

12.) You can swim at a constant speed of 5 mph. You wish to swim across a river 1 mile wide and land at a point directly across the river from where you start swimming. If the river flows at the constant rate of 3 mph, in what direction should you swim to accomplish this ? How long will it take you to swim across the river ?

********** The following problem is for recreational purposes only. **********

13.) A circus is witnessed by 120 people who have paid a total of \$120. The women paid \$5 each, the men paid \$2 each, and the children paid 10 cents each. How many women and children went to the circus ?