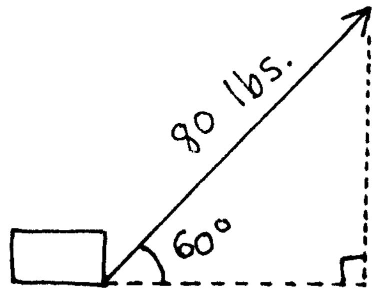
Math 21C
Kouba
Discussion Sheet 6



1.) A force of 80 pounds is applied in the given diagram. Find the horizontal and vertical components of this force.

2.) Let vector $\vec{A} = \overrightarrow{\langle 3, 4 \rangle}$. Find a vector of length 4 which is

a.) parallel to $\vec{A}$.
b.) perpendicular to $\vec{A}$.

3.) Find two unit vectors each of which is perpendicular to both $\vec{A} = \overrightarrow{\langle 1, 0, -2 \rangle}$ and $\vec{B} = \overrightarrow{\langle 0, 3, 4 \rangle}$.

4.) Construct the projection vector $proj_{\vec{B}} \vec{A}$ for each pair of vectors.

a.) $\vec{A} = \overrightarrow{\langle 3, 4 \rangle}$, $\vec{B} = \overrightarrow{\langle -1, 2 \rangle}$
b.) $\vec{A} = \overrightarrow{\langle 1, 2, 3 \rangle}$, $\vec{B} = \overrightarrow{\langle 3, 2, 1 \rangle}$

5.) Determine the area of the triangle formed by the points

a.) $(1, 1), (4, 2)$, and $(-3, 3)$
b.) $(0, 0, 0), (3, -2, 1)$, and $(1, 0, 2)$

6.) Compute the area of the parallelogram formed by the vectors $\vec{A} = \overrightarrow{\langle 4, -1, 2 \rangle}$ and $\vec{B} = \overrightarrow{\langle 2, 3, 0 \rangle}$.

7.) Compute the volume of the parallelepiped formed by the vectors $\vec{u} = \vec{i} + 2\vec{j} - \vec{k}$, $\vec{v} = 2\vec{i} + \vec{j} + 3\vec{k}$, and $\vec{w} = \vec{i} - \vec{j} + 2\vec{k}$.

8.) A jet airplane wants to fly in a straight line from airport A directly East to airport B, which is 1000 miles away. The jet is pushed by a tailwind from 30° South of West at 60 mph. If the jet flies at a constant speed of 300 mph (relative to the surrounding air space),

a.) in what direction should the jet fly ?
b.) what is the jet's actual flying speed (relative to the ground) ?
c.) how long will the flight take ?

9.) Determine parametric equations for the line L passing through the points $(1, -1, 2)$ and $(3, 0, -4)$.

10.) Determine parametric equations for the line L passing through the point $(2, 1, -3)$ and which is parallel to the line M given by

$$M: \begin{cases} x = 1 + t \\ y = 2 - t \\ z = 3t \end{cases}$$

11.) Determine if the following lines intersect. If they do, find the point of intersection and the angle between the lines.

$$L: \begin{cases} x = 3 - t \\ y = 2 + t \\ z = t \end{cases}$$

$$M: \begin{cases} x = 2 + s \\ y = 3 - s \\ z = 2s + 7 \end{cases}$$

12.) Determine parametric equations for the line L representing the intersection of the planes $x + y - 2z = 10$ and $3x + 2y + z = 5$.

13.) Determine the angle between the intersecting planes $x+y-2z = 10$ and $3x+2y+z = 5$.

14.) Consider the plane given by $2x - 4y + 5z = 20$.

a.) Find 3 points lying on this plane.

b.) Find 2 vectors perpendicular (normal) to the plane.

c.) Find a vector parallel to the plane's surface.

15.) Determine an equation of the plane passing through the points $(0, 0, 0)$, $(1, 0, -2)$, and $(0, 3, 4)$.

16.) Compute the distance from the origin to the plane given by $x + 2y + 3z = 6$.

17.) Compute the distance between the parallel planes given by $x + 2y + 3z = 6$ and $x + 2y + 3z = 0$.

18.) Find the point of intersection of the plane given by $x + 2y + 3z = 6$ and the line given by $L: \begin{cases} x = 3 - t \\ y = 2 + t \\ z = t \end{cases}$.

19.) Consider the vectors $\vec{A} = \overrightarrow{(a_1, a_2)}$, $\vec{B} = \overrightarrow{(b_1, b_2)}$, and $\vec{C} = \overrightarrow{(c_1, c_2)}$. Prove that $\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$.

20.) Consider the vectors $\vec{A} = \overrightarrow{(a_1, a_2, a_3)}$, $\vec{B} = \overrightarrow{(b_1, b_2, b_3)}$, and $\vec{C} = \overrightarrow{(c_1, c_2, c_3)}$. Prove that $\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$.

21.) Consider the vectors $\vec{A} = \overrightarrow{(a_1, a_2, a_3)}$ and $\vec{B} = \overrightarrow{(b_1, b_2, b_3)}$. Prove that $|\vec{A} \times \vec{B}|^2 = |\vec{A}|^2 |\vec{B}|^2 - (\vec{A} \cdot \vec{B})^2$.

22.) Consider the vectors $\vec{A} = \overrightarrow{(a_1, a_2, a_3)}$ and $\vec{B} = \overrightarrow{(b_1, b_2, b_3)}$. Prove that $\vec{A} \perp \vec{A} \times \vec{B}$.

23.) Consider the vector $\vec{A} = \overrightarrow{(a_1, a_2, a_3)}$. Prove that

$$\vec{A} \times \vec{A} = \vec{O} \ .$$

************* The following problem is for recreational purposes only. *************

24.) A 12 ft. by 30 ft. room has a 12 ft. ceiling. In the middle of one end wall, one foot above the floor, is a spider. The spider wants to capture a fly in the middle of the opposite end wall, one foot below the ceiling. What is the length of the shortest path the spider can walk (no spider webs allowed) in order to reach the fly ?