Math 21C
Kouba
Discussion Sheet 8

1.) Evaluate the following limits or determine that the limit does not exist.

a.) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2+y^2-4}{x+y+2}$ b.) $\displaystyle\lim_{(x,y)\to(1,1)} \frac{xy-y-2x+2}{x-1}$

c.) $\displaystyle\lim_{(x,y)\to(2,2)} \frac{x+y-4}{\sqrt{x+y}-2}$ d.) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{\sin(x^2+y^2)}{x^2+y^2}$

e.) $\displaystyle\lim_{(x,y)\to(1,1)} \frac{\sin(x^2-y^2)}{x-y}$ f.) $\displaystyle\lim_{(x,y)\to(1,-1)} \arcsin \frac{xy}{\sqrt{x^2+y^2}}$

g.) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3}{x^3+y^3}$ h.) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$ i.) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4}$

j.) $\displaystyle\lim_{(x,y)\to(2,-2)} \frac{4-xy}{4+xy}$ k.) $\displaystyle\lim_{(x,y)\to(0,0)} (1+3xy^2)^{2/xy^2}$ l.) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4}$

m.) $\displaystyle\lim_{(x,y)\to(1,-2)} \frac{(x-1)^2+3(y+2)^2}{x-1+(y+2)^2}$ n.) $\displaystyle\lim_{(x,y)\to(1,2)} \frac{xy+2x-y-2}{xy-y+3x-3}$

2.) Compute z_x and z_y for each of the following functions.

a.) $z = xy^2 + \ln x + e^y + 5$ b.) $z = xe^{2y} \arctan x$ c.) $z = \sqrt{x-y^2}$

d.) $z = \dfrac{x^3}{y^2} + \sin(xy)$ e.) $z = \dfrac{x+4}{x^2+y^2}$ f.) $z = \{e^{x^2 y} + \tan(3y+4x)\}^5$

f.) $z = y^{1+x^3}$

3.) Show that $z = \ln(1+x^2+y^2)$ satisfies the equation $z_{xy} + z_x z_y = 0$.

4.) Verify that $w_{xy} = w_{yx}$ for $w = y + \dfrac{x}{y}$.

5.) Determine functions z whose partial derivatives are given, or state that this is impossible.

a.) $z_x = 2x$ and $z_y = 3y^2+1$ b.) $z_x = xy^2 - y$ and $z_y = x^2 y - x$

c.) $z_x = e^x y - 1$ and $z_y = e^x - x$

d.) $z_x = ye^x \cos(xy) + e^x \sin(xy) - 2$ and $z_y = xe^x \cos(xy) + 1$

6.) Consider the function $f(x,y) = \begin{cases} \dfrac{\sin(x^3+y)}{x^2+y^2}, & \text{if } (x,y) \neq (0,0) \\ 0 \quad , & \text{if } (x,y) = (0,0) \end{cases}$.

a.) Determine $f_x(x,y)$ when $(x,y) \neq (0,0)$.

b.) Determine $f_x(0,0)$ (Use limit definition of partial derivative.).

c.) Determine $f_y(0,0)$ (Use limit definition of partial derivative.).

7.) Plane A, parallel to the xz-plane, and plane B, parallel to the yz-plane, pass through the surface determined by the equation $z = xy^2 - x^3 + 7$. Both planes include the point

$(1, 0, 6)$, which lies on the surface.

a.) Determine the slope of the line tangent to the surface at the point $(1, 0, 6)$ if the line lies in

i.) plane A.

ii.) plane B.

b.) Determine an equation of the plane tangent to the surface at the point $(1, 0, 6)$.

8.) Compute z_x and z_y for each of the following functions.

a.) $z = x^3y + y^4 - 2x + 5$ b.) $z = f(x) + g(y)$ c.) $z = f(x^3) + g(4y)$

d.) $z = f(x^2 + y^3) + g(xy^2)$ e.) $y^2 + z^2 + \sin(xz) = 4$

f.) $z = f(u, v)$ where $u = \ln(x - y)$ and $v = e^{xy}$

9.) Find $\dfrac{\partial w}{\partial t}$ and $\dfrac{\partial w}{\partial s}$ if $w = f(4t^2 - 3s)$ and $f'(x) = \ln x$.

10.) Assume that f is differentiable function of one variable with $z = xf(xy)$. Show that $xz_x - yz_y = z$.

11.) Assume that f and g are twice differentiable functions of one variable. Show that $u = f(x + at) + g(x - at)$ satisfies $a^2 \dfrac{\partial^2 u}{\partial x^2} = \dfrac{\partial^2 u}{\partial t^2}$, where a is a constant.

12.) Consider the paraboloid given by $f(x, y) = 25 - x^2 - y^2$.

a.) Sketch the surface.

b.) Let point $P = (2, -2)$. Compute the derivative of the function f at the point P in the direction

i.) $\overrightarrow{A} = \overrightarrow{(-3, 4)}$

ii.) $\overrightarrow{A} = \overrightarrow{(3, -4)}$

iii.) $\overrightarrow{A} = \overrightarrow{(1, 0)}$

iv.) $\overrightarrow{A} = \overrightarrow{(0, -1)}$

c.) In what directions is the derivative of f at point $P = (2, -2)$ equal to zero ?

d.) In what directions is the derivative of f at point $P = (-1, 1)$ equal to 2 ?

THE FOLLOWING PROBLEM IS FOR RECREATIONAL PURPOSES ONLY.

13.) Determine the exact value of the "continued" square root :

$$\sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \cdots}}}}$$