

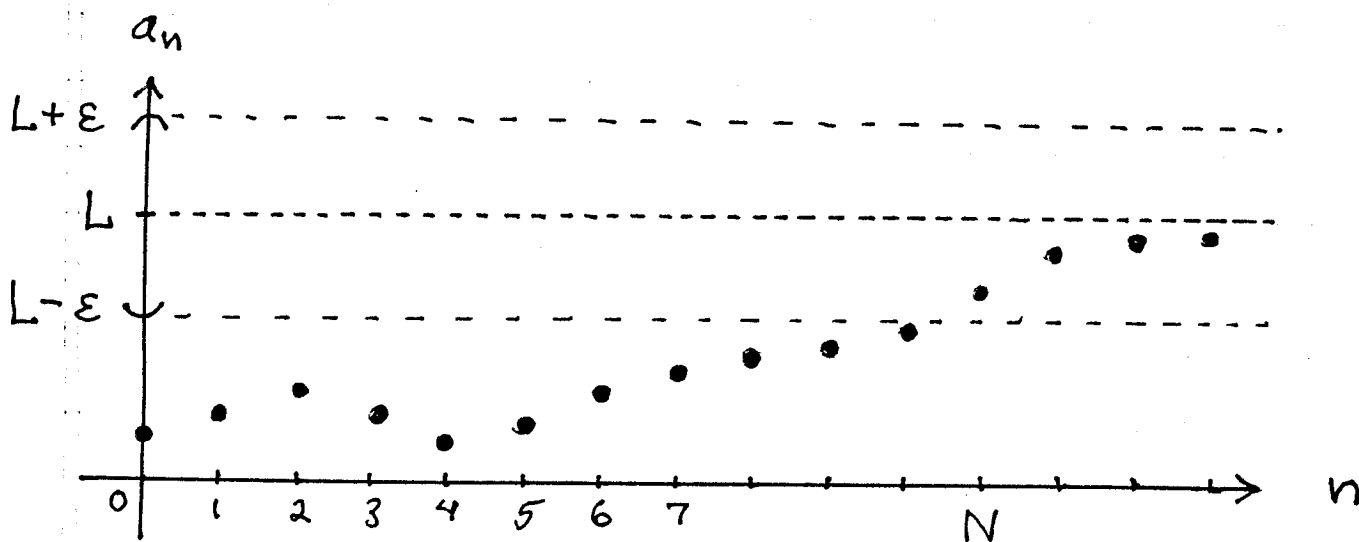
Math 21C

Kouba

Formal Definition of Limit of a Sequence

Definition : $\lim_{n \rightarrow \infty} a_n = L$ (a finite #)

means : For each number $\varepsilon > 0$
there exists an integer N so that
if $n > N$, then $|a_n - L| < \varepsilon$.



Note : $|a_n - L| < \varepsilon \rightarrow$

$$-\varepsilon < a_n - L < \varepsilon \rightarrow$$

$$L - \varepsilon < a_n < L + \varepsilon$$

Example : Prove that $\lim_{n \rightarrow \infty} \frac{n}{n+2} = 1$.

Proof : Let $\varepsilon > 0$ be given. Find an integer N so that

$$\text{if } n > N, \text{ then } \left| \frac{n}{n+2} - 1 \right| < \varepsilon.$$

Begin with $\left| \frac{n}{n+2} - 1 \right| < \varepsilon$ and solve for n . Then

$$\left| \frac{n}{n+2} - 1 \right| < \varepsilon \text{ iff } \left| \frac{n}{n+2} - \frac{n+2}{n+2} \right| < \varepsilon$$

$$\text{iff } \left| \frac{n - n - 2}{n+2} \right| < \varepsilon$$

$$\text{iff } \left| \frac{-2}{n+2} \right| < \varepsilon$$

$$\text{iff } \frac{|-2|}{|n+2|} < \varepsilon$$

$$\text{iff } \frac{2}{|n+2|} < \varepsilon$$

$$\text{iff } \frac{2}{n+2} < \varepsilon \quad (\text{Since } n \rightarrow \infty, n \text{ is } +)$$

$$\text{iff } \frac{2}{\varepsilon} < n+2 \quad \text{iff } n > \frac{2}{\varepsilon} - 2.$$

Now CHOOSE N to be any integer

$N \geq \frac{2}{\varepsilon} - 2$. Thus, if $n > N$, it

follows that $\left| \frac{n}{n+2} - 1 \right| < \varepsilon$. QED