

Section 11.2

$$\begin{aligned}
 7.) \quad \sum_{n=0}^{\infty} \frac{(-1)^n}{4^n} &= 1 + \frac{-1}{4} + \frac{1}{4^2} + \frac{-1}{4^3} + \frac{1}{4^4} + \frac{-1}{4^5} + \dots \\
 &= 1 + \left(\frac{-1}{4}\right) + \left(\frac{-1}{4}\right)^2 + \left(\frac{-1}{4}\right)^3 + \left(\frac{-1}{4}\right)^4 + \dots \\
 &= \frac{1}{1 - \left(\frac{-1}{4}\right)} = \frac{1}{5/4} = \frac{4}{5}
 \end{aligned}$$

$$\begin{aligned}
 8.) \quad \sum_{n=2}^{\infty} \frac{1}{4^n} &= \frac{1}{4^2} + \frac{1}{4^3} + \frac{1}{4^4} + \frac{1}{4^5} + \dots \\
 &= \frac{1}{4^2} \cdot \left[1 + \left(\frac{1}{4}\right) + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^3 + \dots \right] \\
 &= \frac{1}{16} \cdot \frac{1}{1 - \left(\frac{1}{4}\right)} = \frac{1}{16} \cdot \frac{4}{3} = \frac{1}{12}
 \end{aligned}$$

$$\begin{aligned}
 12.) \quad \sum_{n=0}^{\infty} \left(\frac{5}{2^n} - \frac{1}{3^n} \right) &= 5 \sum_{n=0}^{\infty} \frac{1}{2^n} - \sum_{n=0}^{\infty} \frac{1}{3^n} \\
 &= 5 \left(1 + \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 + \dots \right) - \left(1 + \left(\frac{1}{3}\right) + \left(\frac{1}{3}\right)^2 + \dots \right) \\
 &= 5 \cdot \frac{1}{1 - \left(\frac{1}{2}\right)} - \frac{1}{1 - \left(\frac{1}{3}\right)} \\
 &= 5 \cdot (2) - \frac{3}{2} = \frac{17}{2}
 \end{aligned}$$

$$\begin{aligned}
 14.) \quad \sum_{n=0}^{\infty} \left(\frac{2^{n+1}}{5^n} \right) &= \sum_{n=0}^{\infty} 2 \cdot \frac{2^n}{5^n} \\
 &= 2 \cdot \left[1 + \left(\frac{2}{5}\right) + \left(\frac{2}{5}\right)^2 + \left(\frac{2}{5}\right)^3 + \dots \right] \\
 &= 2 \cdot \frac{1}{1 - \frac{2}{5}} = 2 \cdot \frac{5}{3} = \frac{10}{3}
 \end{aligned}$$

$$16.) \quad \frac{6}{(2n-1)(2n+1)} = \frac{A}{2n-1} + \frac{B}{2n+1}$$

$$= \frac{A(2n+1) + B(2n-1)}{(2n-1)(2n+1)} \rightarrow$$

$$A(2n+1) + B(2n-1) = 6$$

$$\text{Let } x = \frac{1}{2} : 2A = 6 \rightarrow A = 3$$

$$\text{Let } x = -\frac{1}{2} : -2B = 6 \rightarrow B = -3 ; \text{ then}$$

$$\sum_{n=1}^{\infty} \frac{6}{(2n-1)(2n+1)} = \sum_{n=1}^{\infty} \left(\frac{3}{2n-1} + \frac{-3}{2n+1} \right) ;$$

$$S_1 = 3 + (-1) = 2 ,$$

$$S_2 = (3 + (-1)) + (1 + (-\frac{3}{5})) = 3 - \frac{3}{5} ,$$

$$S_3 = (3 + (-1)) + (1 + (-\frac{3}{5})) + (\frac{3}{5} + (-\frac{3}{7})) = 3 - \frac{3}{7} ,$$

$$S_4 = (3 + (-1)) + (1 + (-\frac{3}{5})) + (\frac{3}{5} + (-\frac{3}{7})) + (\frac{3}{7} + (-\frac{3}{9})) = 3 - \frac{3}{9} ,$$

⋮

$$S_n = 3 - \frac{3}{2n+1} ; \text{ and by sequence of partial sums series converges since}$$

$$\sum_{n=1}^{\infty} \frac{6}{(2n-1)(2n+1)} = \lim_{n \rightarrow \infty} \left(3 - \frac{3}{2n+1} \right) = 3 - 0 = 3$$

$$19.) \sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}} \right) ;$$

$$S_1 = 1 - \frac{1}{\sqrt{2}} ,$$

$$S_2 = (1 - \frac{1}{\sqrt{2}}) + (\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}}) = 1 - \frac{1}{\sqrt{3}} ,$$

$$S_3 = (1 - \frac{1}{\sqrt{2}}) + (\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}}) + (\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}}) = 1 - \frac{1}{\sqrt{4}} ,$$

\vdots
 $S_n = 1 - \frac{1}{\sqrt{n+1}}$; and by sequence
of partial sums series converges since

$$\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}} \right) = \lim_{n \rightarrow \infty} \left(1 - \frac{1}{\sqrt{n+1}} \right)$$

$$= 1 - 0 = 1.$$

24.) $\sum_{n=0}^{\infty} (\sqrt{2})^n$; $\lim_{n \rightarrow \infty} (\sqrt{2})^n = \infty \neq 0$

so series diverges by nth-term test.

25.) $\sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{3}{2^n} = \sum_{n=1}^{\infty} (-1)^n \cdot (-1) \cdot \frac{3}{2^n}$
 $= -3 \cdot \sum_{n=1}^{\infty} \left(\frac{-1}{2} \right)^n$ so series converges

by geometric series test since $r = -\frac{1}{2}$ and $-1 < r < 1$ with value

$$-3 \sum_{n=1}^{\infty} \left(\frac{-1}{2} \right)^n = -3 \cdot \left[\left(\frac{-1}{2} \right) + \left(\frac{-1}{2} \right)^2 + \left(\frac{-1}{2} \right)^3 + \dots \right]$$

$$= (-3) \left(\frac{-1}{2} \right) \cdot \left[1 + \left(\frac{-1}{2} \right) + \left(\frac{-1}{2} \right)^2 + \left(\frac{-1}{2} \right)^3 + \dots \right]$$

$$= \frac{3}{2} \cdot \frac{1}{1 - \left(\frac{-1}{2} \right)} = \frac{3}{2} \cdot \frac{2}{3} = 1$$

26.) $\sum_{n=1}^{\infty} (-1)^{n+1} \cdot n$; $\lim_{n \rightarrow \infty} (-1)^{n+1} \cdot n \neq 0$

so series diverges by nth-term test.

$$\begin{aligned}
 28.) \sum_{n=0}^{\infty} \frac{\cos 2n\pi}{5^n} &= \frac{\cos 0}{1} + \frac{\cos 2\pi}{5} + \frac{\cos 4\pi}{5^2} + \dots \\
 &= 1 - \frac{1}{5} + \frac{1}{5^2} - \frac{1}{5^3} + \dots \\
 &= 1 + \left(-\frac{1}{5}\right) + \left(-\frac{1}{5}\right)^2 + \left(-\frac{1}{5}\right)^3 + \dots \\
 &= \frac{1}{1 - \left(-\frac{1}{5}\right)} = \frac{5}{6} \quad \text{so series converges} \\
 &\quad \text{by geometric} \\
 &\quad \text{series test since } r = -\frac{1}{5} \\
 &\quad \text{and } -1 < r < 1.
 \end{aligned}$$

$$30.) \sum_{n=1}^{\infty} \ln\left(\frac{1}{n}\right); \quad \lim_{n \rightarrow \infty} \ln\left(\frac{1}{n}\right) = -\infty \neq 0$$

so series diverges by nth-term test

$$34.) \sum_{n=1}^{\infty} \left(1 - \frac{1}{n}\right)^n; \quad \lim_{n \rightarrow \infty} \left(1 - \frac{1}{n}\right)^n$$

$$= \lim_{n \rightarrow \infty} \left(1 + \frac{(-1)}{n}\right)^n = e^{-1} \neq 0 \quad \text{so}$$

series diverges by nth-term test

$$36.) \sum_{n=1}^{\infty} \frac{n^n}{n!} = \frac{1}{1} + \frac{2 \cdot 2}{2 \cdot 1} + \frac{3 \cdot 3 \cdot 3}{3 \cdot 2 \cdot 1} + \frac{4 \cdot 4 \cdot 4 \cdot 4}{4 \cdot 3 \cdot 2 \cdot 1} + \dots$$

$> 1 + 1 + 1 + 1 + \dots$ so series

diverges by comparison to a divergent series

$$37.) \sum_{n=1}^{\infty} \ln \left(\frac{n}{n+1} \right) = \sum_{n=1}^{\infty} (\ln n - \ln(n+1));$$

$$S_1 = \ln 1 - \ln 2 = -\ln 2$$

$$S_2 = (\ln 1 - \ln 2) + (\ln 2 - \ln 3) = -\ln 3,$$

$$S_3 = (\ln 1 - \ln 2) + (\ln 2 - \ln 3) + (\ln 3 - \ln 4)$$

$$= -\ln 4,$$

⋮

$S_n = -\ln(n+1)$; and by sequence of partial sums series diverges since

$$\sum_{n=1}^{\infty} \ln \left(\frac{n}{n+1} \right) = \lim_{n \rightarrow \infty} -\ln(n+1) = -\infty$$

$$39.) \sum_{n=0}^{\infty} \left(\frac{e}{\pi} \right)^n = 1 + \left(\frac{e}{\pi} \right) + \left(\frac{e}{\pi} \right)^2 + \left(\frac{e}{\pi} \right)^3 + \dots$$

$$= \frac{1}{1 - \left(\frac{e}{\pi} \right)} = \frac{\pi}{\pi - e} \quad \text{so series}$$

converges by geometric series test since $r = \frac{e}{\pi}$ and $-1 < r < 1$.

$$52.) 0.234234234\dots$$

$$= \frac{234}{1000} + \frac{234}{1,000,000} + \frac{234}{1,000,000,000} + \dots$$

$$= 234 \left(\frac{1}{10^3} + \frac{1}{10^6} + \frac{1}{10^9} + \frac{1}{10^{12}} + \dots \right)$$

$$= (234) \left(\frac{1}{10^3} \right) \cdot \left[1 + \frac{1}{10^3} + \frac{1}{10^6} + \frac{1}{10^9} + \dots \right]$$

$$\begin{aligned}
&= \frac{234}{1000} \cdot \left[1 + \left(\frac{1}{1000}\right) + \left(\frac{1}{1000}\right)^2 + \left(\frac{1}{1000}\right)^3 + \dots \right] \\
&= \frac{234}{1000} \cdot \frac{1}{1 - \frac{1}{1000}} = \frac{234}{999} = \frac{26}{111}
\end{aligned}$$

53.) $0.7777\dots$

$$= \frac{7}{10} + \frac{7}{100} + \frac{7}{1000} + \frac{7}{10,000} + \dots$$

$$= \frac{7}{10} \cdot \left[1 + \left(\frac{1}{10}\right) + \left(\frac{1}{10}\right)^2 + \left(\frac{1}{10}\right)^3 + \dots \right]$$

$$= \frac{7}{10} \cdot \frac{1}{1 - \frac{1}{10}} = \frac{7}{10} \cdot \frac{10}{9} = \frac{7}{9}$$

66.) If $\sum a_n$ converges and $a_n > 0$, then $\lim_{n \rightarrow \infty} a_n = 0$. Thus,

$$\lim_{n \rightarrow \infty} \frac{1}{a_n} = \frac{1}{0^+} = +\infty \neq 0 \text{ so}$$

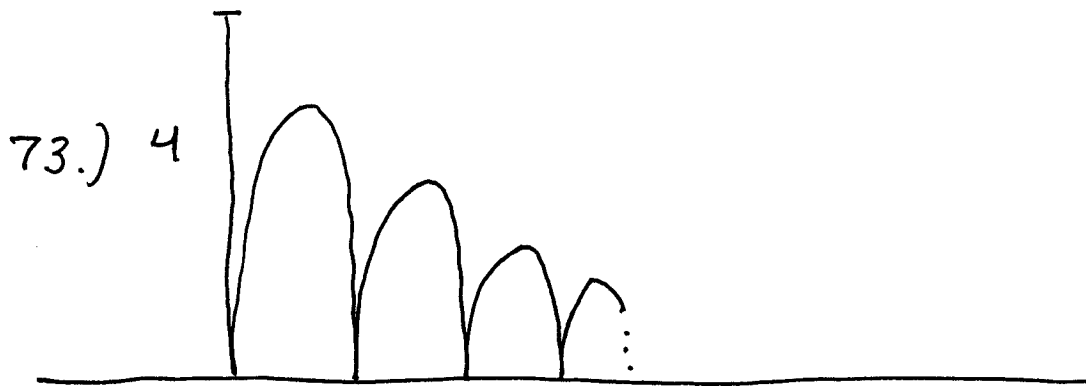
$\sum \frac{1}{a_n}$ diverges by the n th-term test.

70.) $1 + e^b + e^{2b} + e^{3b} + \dots = 9 \rightarrow$

$$1 + (e^b) + (e^b)^2 + (e^b)^3 + \dots = 9 \rightarrow$$

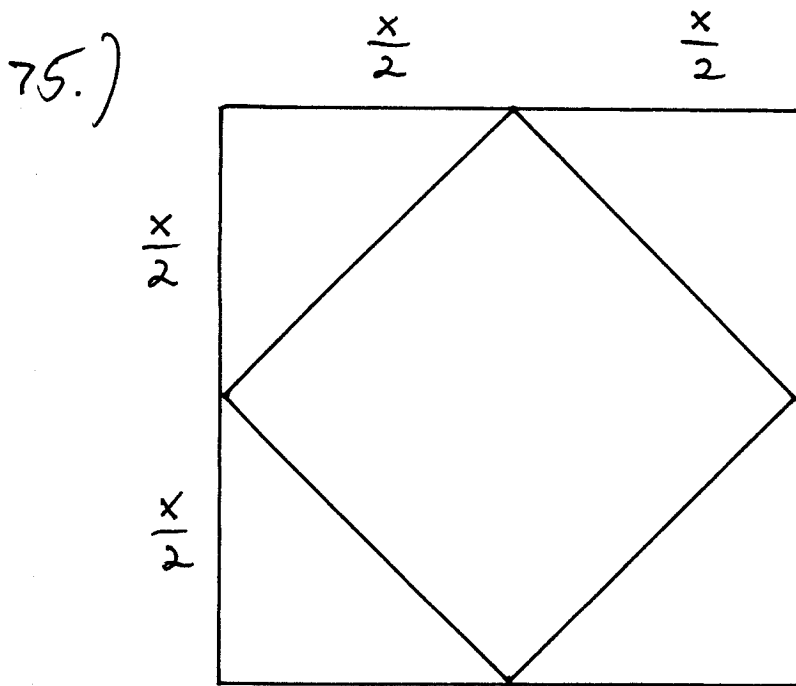
$$\frac{1}{1 - e^b} = 9 \rightarrow \frac{1}{9} = 1 - e^b \rightarrow$$

$$e^b = \frac{8}{9} \rightarrow b = \ln\left(\frac{8}{9}\right).$$



Total distance is

$$\begin{aligned}
 T &= 4 + 2(0.75)(4) + 2(0.75)^2(4) + 2(0.75)^3(4) + \dots \\
 &= 4 + (2)(0.75)(4) \cdot [1 + (0.75) + (0.75)^2 + \dots] \\
 &= 4 + (6) \cdot \frac{1}{1 - (0.75)} \\
 &= 4 + (6) \cdot (4) = 28 \text{ ft.}
 \end{aligned}$$



assume big square is x by x , then area of inscribed square is

$$\begin{aligned}
 \text{Area} &= x^2 - 4 \left(\frac{x}{2}\right)^2 \cdot \frac{1}{2} = \frac{1}{2} x^2 ; \\
 \text{inscribed square is } &\underline{\underline{\frac{1}{\sqrt{2}} x \text{ by } \frac{1}{\sqrt{2}} x}} ;
 \end{aligned}$$

then sum of areas of all squares
is

$$\begin{aligned} S &= (2)^2 + \left(\frac{1}{\sqrt{2}} \cdot 2\right)^2 + \left(\left(\frac{1}{\sqrt{2}}\right)^2 \cdot 2\right)^2 \\ &\quad + \left(\left(\frac{1}{\sqrt{2}}\right)^3 \cdot 2\right)^2 + \left(\left(\frac{1}{\sqrt{2}}\right)^4 \cdot 2\right)^2 + \dots \\ &= 4 + \left(\frac{1}{\sqrt{2}}\right)^2 \cdot (4) + \left(\frac{1}{\sqrt{2}}\right)^4 \cdot (4) + \left(\frac{1}{\sqrt{2}}\right)^6 \cdot (4) + \dots \\ &= 4 \cdot \left[1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots\right] \\ &= 4 \cdot \left[1 + \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \dots\right] \\ &= 4 \cdot \frac{1}{1 - \left(\frac{1}{2}\right)} = 8 \text{ m}^2 \end{aligned}$$