

Math 21C

Kouba

Plane Vectors, Space Vectors

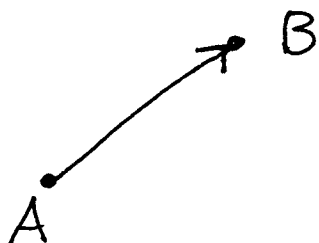
Def: A vector is a symbol representing direction and magnitude (length)

Ex: Vectors can describe

- 1.) speed and direction of fluid flow
- 2.) speed and direction of object moving along a path
- 3.) magnitude and direction of an applied force

Note: Two vectors are equal if they have the same direction and magnitude.

Notation: Vector $\overrightarrow{AB}$ is a vector with initial point A and terminal point B.



Ex: Find vector $\overrightarrow{AB}$.

- 1.) $A: (1, 1)$, $B: (4, 3)$
- 2.) $A: (3, -1)$, $B: (-2, -4)$

$$3.) A: (0, 2), B: (3, 4)$$

$$4.) A: (1, 0, -1), B: (2, -3, 4)$$

$$5.) A: (3, -2, 2), B: (0, 0, 0)$$

Def: Let $(\overrightarrow{a, b})$ and $(\overrightarrow{a, b, c})$ be vectors.
The magnitude of each vector is

$$1.) |\overrightarrow{(a, b)}| = \sqrt{a^2 + b^2}$$

$$2.) |\overrightarrow{(a, b, c)}| = \sqrt{a^2 + b^2 + c^2}$$

Algebra of Vectors:

Let k be a constant.

$$1.) a.) (\overrightarrow{a, b}) + (\overrightarrow{c, d}) = (\overrightarrow{a+c, b+d})$$

$$b.) k(\overrightarrow{a, b}) = (\overrightarrow{ka, kb})$$

$$2.) a.) (\overrightarrow{a, b, c}) + (\overrightarrow{d, e, f}) = (\overrightarrow{a+d, b+e, c+f})$$

$$b.) k(\overrightarrow{a, b, c}) = (\overrightarrow{ka, kb, kc})$$

$$3.) a.) \text{ If } (\overrightarrow{a, b}) = (\overrightarrow{c, d}), \text{ then } a=c \text{ and } b=d.$$

$$b.) \text{ If } (\overrightarrow{a, b, c}) = (\overrightarrow{d, e, f}), \text{ then } a=d, b=e, \text{ and } c=f.$$

$$4.) a.) |k(\overrightarrow{a, b})| = |k| |\overrightarrow{(a, b)}|$$

$$b.) |k(\overrightarrow{a, b, c})| = |k| |\overrightarrow{(a, b, c)}|$$

Def: any vector $\vec{u}$ of length 1 is called a unit vector.

Ex: Show that each vector is a unit vector.

1.) $\overrightarrow{(\frac{1}{2}, -\frac{\sqrt{3}}{2})}$ 2.) $\overrightarrow{(-1, 0)}$

3.) $\overrightarrow{(-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}})}$ 4.) $\overrightarrow{(\frac{1}{3}, -\frac{2}{3}, \frac{2}{3})}$

Fact: Let $\vec{w}$ be any vector.

A unit vector in the same direction as $\vec{w}$ is

$$\vec{u} = \frac{1}{|\vec{w}|} \vec{w}$$

Ex: Make each vector a unit vector in the same direction.

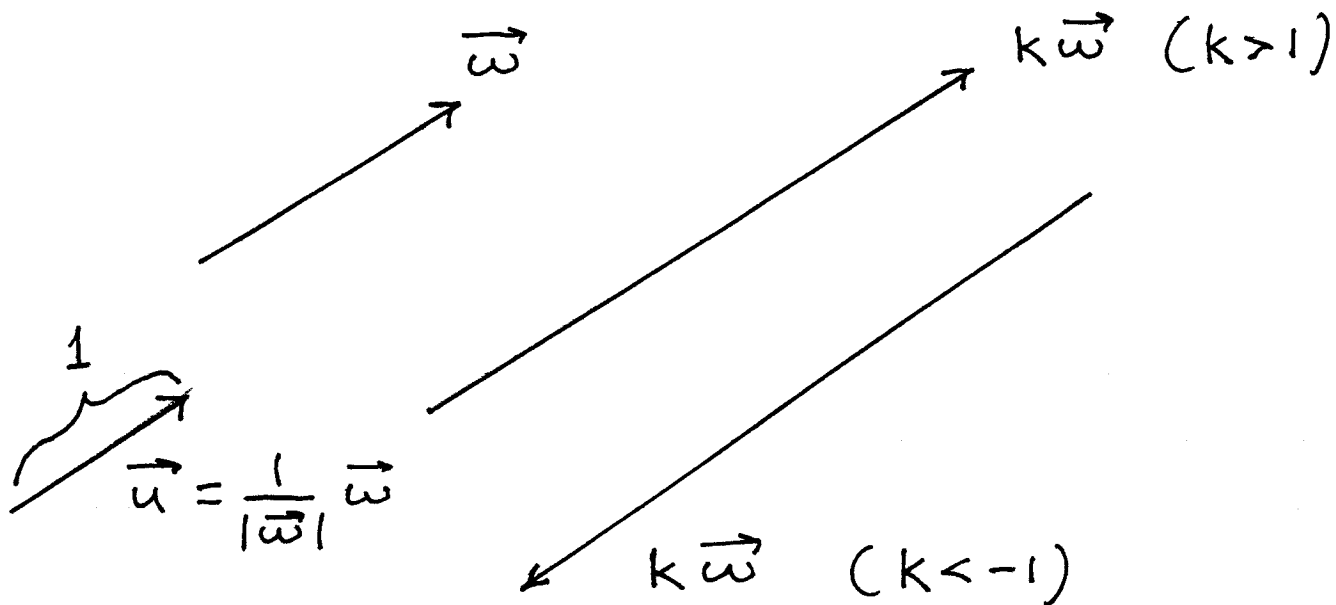
1.) $\overrightarrow{(1, -1)}$ 2.) $\overrightarrow{(3, 4)}$

3.) $\overrightarrow{(0, 0, 3)}$ 4.) $\overrightarrow{(-2, 3, -6)}$

Geometry of Vectors

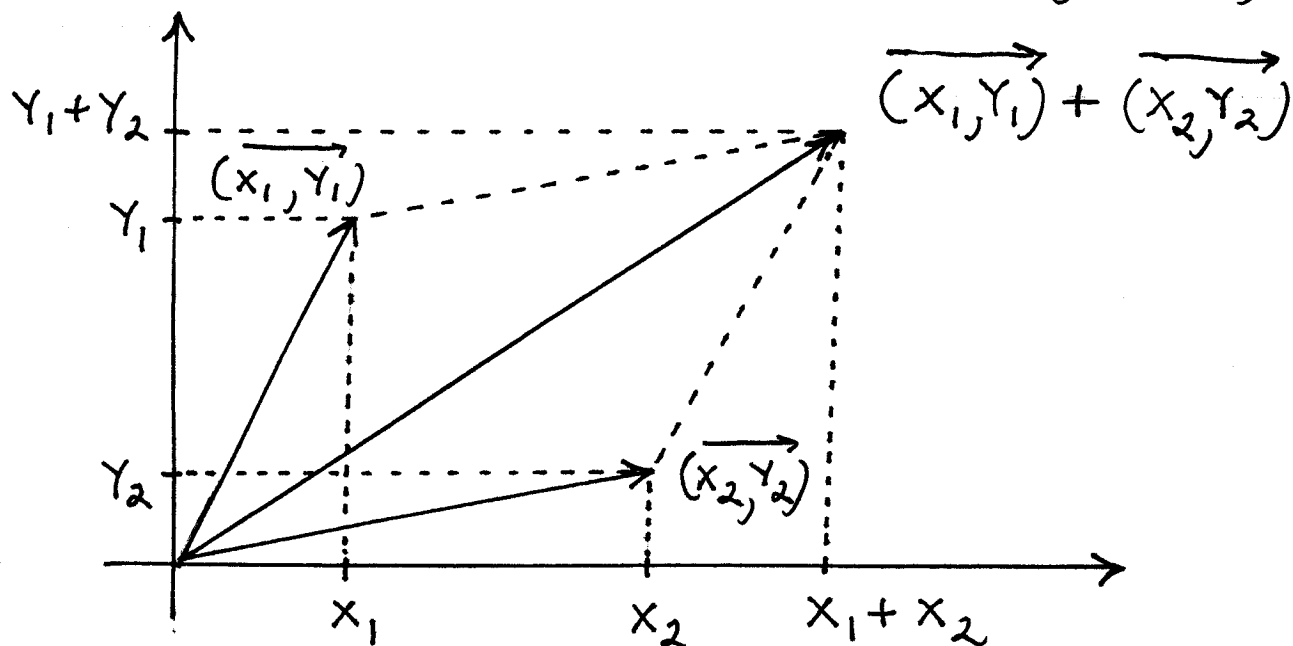
I.) Scalar Multiplication:

Let $\vec{w}$ be a vector, k a constant



II.) Vector Addition :

$$\overrightarrow{(x_1, y_1)} + \overrightarrow{(x_2, y_2)} = \overrightarrow{(x_1 + x_2, y_1 + y_2)}$$



III.) Vector Subtraction :

$$\begin{aligned} \overrightarrow{(x_1, y_1)} - \overrightarrow{(x_2, y_2)} &= \overrightarrow{(x_1, y_1)} + -\overrightarrow{(x_2, y_2)} \\ &= \overrightarrow{(x_1 - x_2, y_1 - y_2)} \end{aligned}$$

