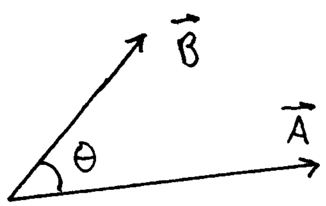
Math 21C

Kouba

Why Does Right Hand Rule for $\vec{A} \times \vec{B}$ work?

<u>Justification</u>: Let $\vec{A} = \overrightarrow{(a_1, a_2, a_3)}$ and $\vec{B} = \overrightarrow{(b_1, b_2, b_3)}$ and let $0 < \theta < 180°$ be the angle between them.



"Translate" vectors $\vec{A}$ and $\vec{B}$ to the xy-plane in such a way that :

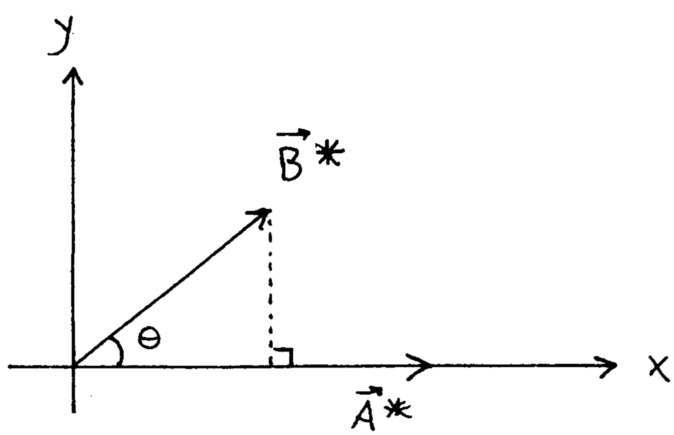
i.) their tails are at the origin,

ii.) $\vec{A}$ points in the direction of $\vec{i}$,

iii.) their lengths are preserved,

and iv.) the angle between them remains θ.

Call the translated vectors $\vec{A}^*$ and $\vec{B}^*$.



It follows that $\vec{A}^* = \overrightarrow{(a, 0, 0)}$ and $\vec{B}^* = \overrightarrow{(b\cos\theta, b\sin\theta, 0)}$

$a = |\vec{A}^*| = |\vec{A}| > 0$ and $b = |\vec{B}^*| = |\vec{B}| > 0$. Then

$$\vec{A}^* \times \vec{B}^* = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a & 0 & 0 \\ b\cos\theta & b\sin\theta & 0 \end{vmatrix} = (ab\sin\theta)\,\vec{k}$$

where $ab\sin\theta > 0$. This vector points in the direction of the positive z-axis, consistent with the Right Hand Rule. □