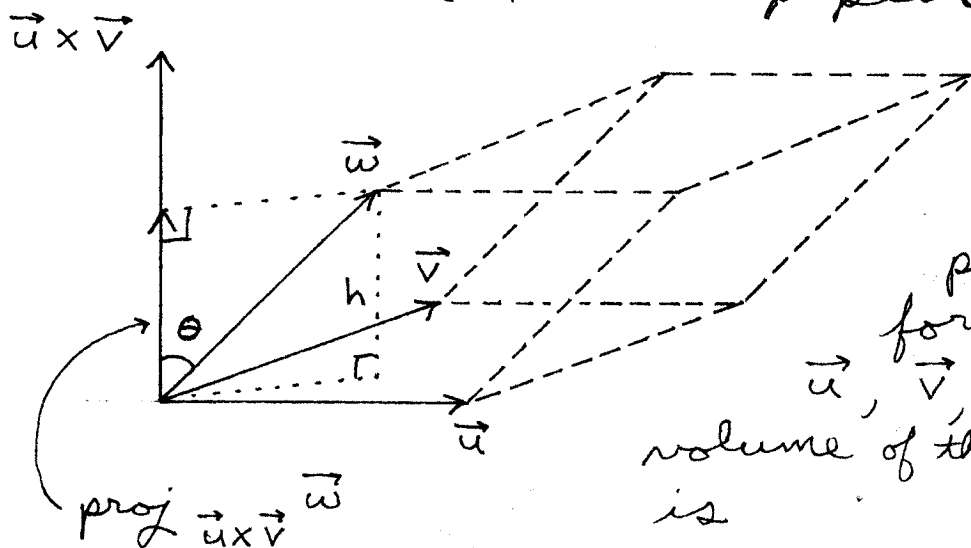


Math 21C

Kouba

Triple Scalar Products and Parallelepipeds



Consider the given parallelepiped formed by vectors $\vec{u}$, $\vec{v}$, and $\vec{w}$. The volume of the parallelepiped is

$$\text{Volume} = (\text{area of base})(\text{height})$$

$$= |\vec{u} \times \vec{v}| \cdot h$$

$$= |\vec{u} \times \vec{v}| \cdot |\text{proj}_{\vec{u} \times \vec{v}} \vec{w}|$$

$$= |\vec{u} \times \vec{v}| \cdot |\vec{w}| |\cos \theta|$$

$$= |(\vec{u} \times \vec{v}) \cdot \vec{w}| ;$$

we call $(\vec{u} \times \vec{v}) \cdot \vec{w}$ the triple scalar product of $\vec{u}$, $\vec{v}$, and $\vec{w}$. The following facts are easily proven :

$$1.) (\vec{u} \times \vec{v}) \cdot \vec{w} = (\vec{v} \times \vec{w}) \cdot \vec{u} = (\vec{w} \times \vec{u}) \cdot \vec{v}$$

$$2.) (\vec{u} \times \vec{v}) \cdot \vec{w} = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$