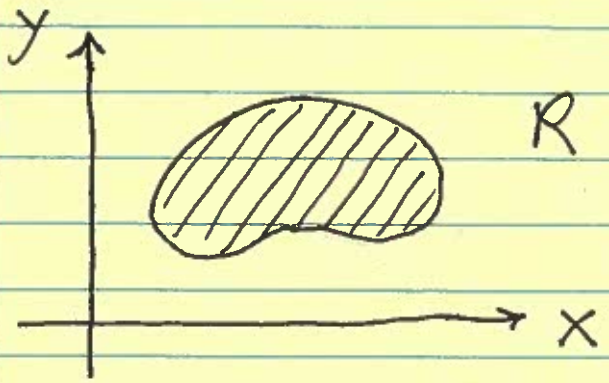
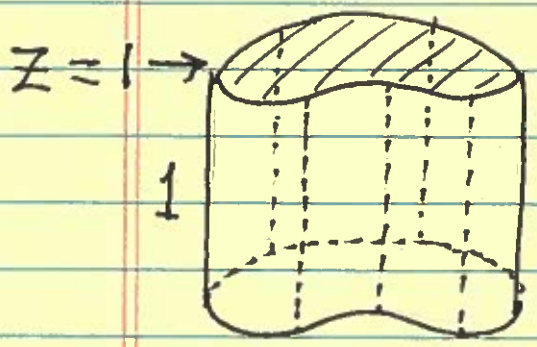
Section 15.2
Thomas Calculus
11th Ed.

Area and Double Integrals



Consider region R in the xy-plane.

Consider the solid of HEIGHT 1 directly above region R:



The VOLUME of this solid is

VOL = (area base)(height)

= (area base)(1)

= area base, i.e.

$$\iint_R 1 \, dA = \text{VOL}$$

= area base

= area of R, i.e.,

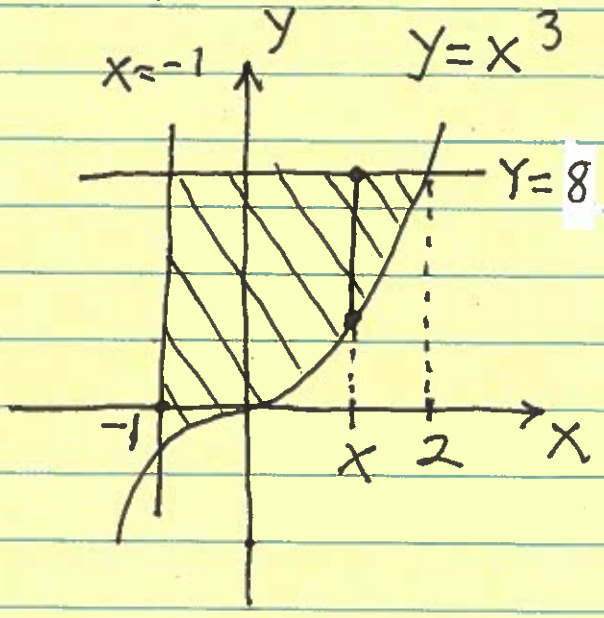
$$\text{AREA of } R = \iint_R 1 \, dA$$

Example: Use a Double Integral to find the AREA of each region R, which is bounded by the graphs of the given equations. (SET UP ONLY)

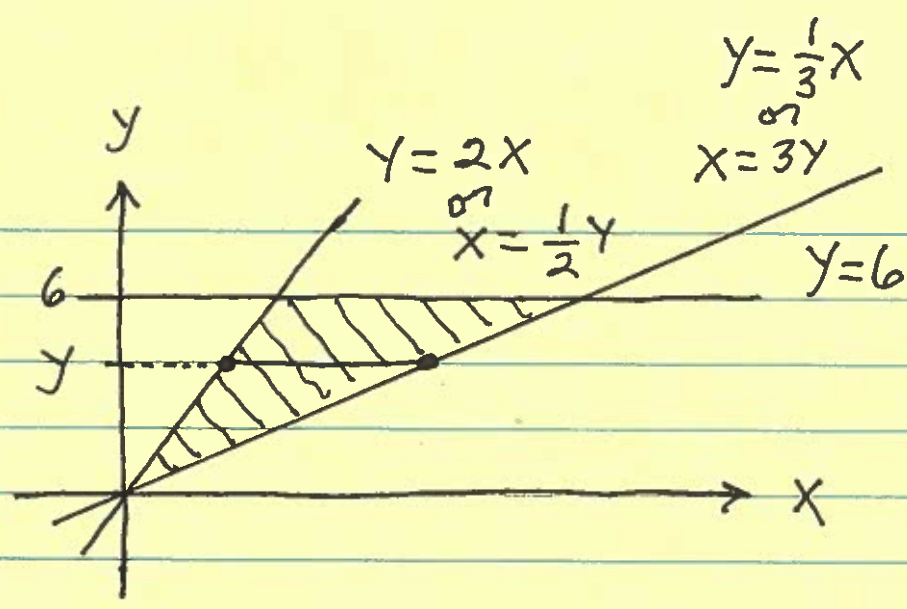
1.) $R: y = x^3, \; y = 8, \; x = -1$

$$R: \begin{cases} -1 \le x \le 2 \\ x^3 \le y \le 8 \end{cases}$$

$$\text{AREA} = \int_{-1}^{2} \int_{x^3}^{8} 1 \, dy \, dx$$



2.) $R: y = 2x, \; y = \frac{1}{3}x, \; y = 6$



$$R:\begin{cases} 0 \le y \le 6 \\ \frac{1}{2}y \le x \le 3y \end{cases}$$

$$AREA = \int_0^6 \int_{\frac{1}{2}y}^{3y} 1 \, dx \, dy$$

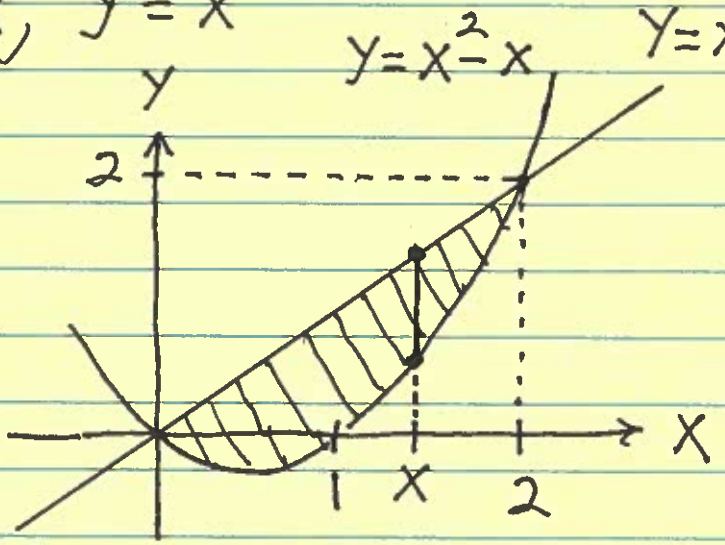
3.) $R: \; y = x^2 - x, \; y = x$

$x^2 - x = x \rightarrow$

$x^2 - 2x = 0 \rightarrow$

$x(x-2) = 0 \rightarrow$

$x = 0, \; x = 2$



$$R:\begin{cases} 0 \le x \le 2 \\ x^2 - x \le y \le x \end{cases}$$

$$AREA = \int_0^2 \int_{x^2-x}^{x} 1 \, dy \, dx$$

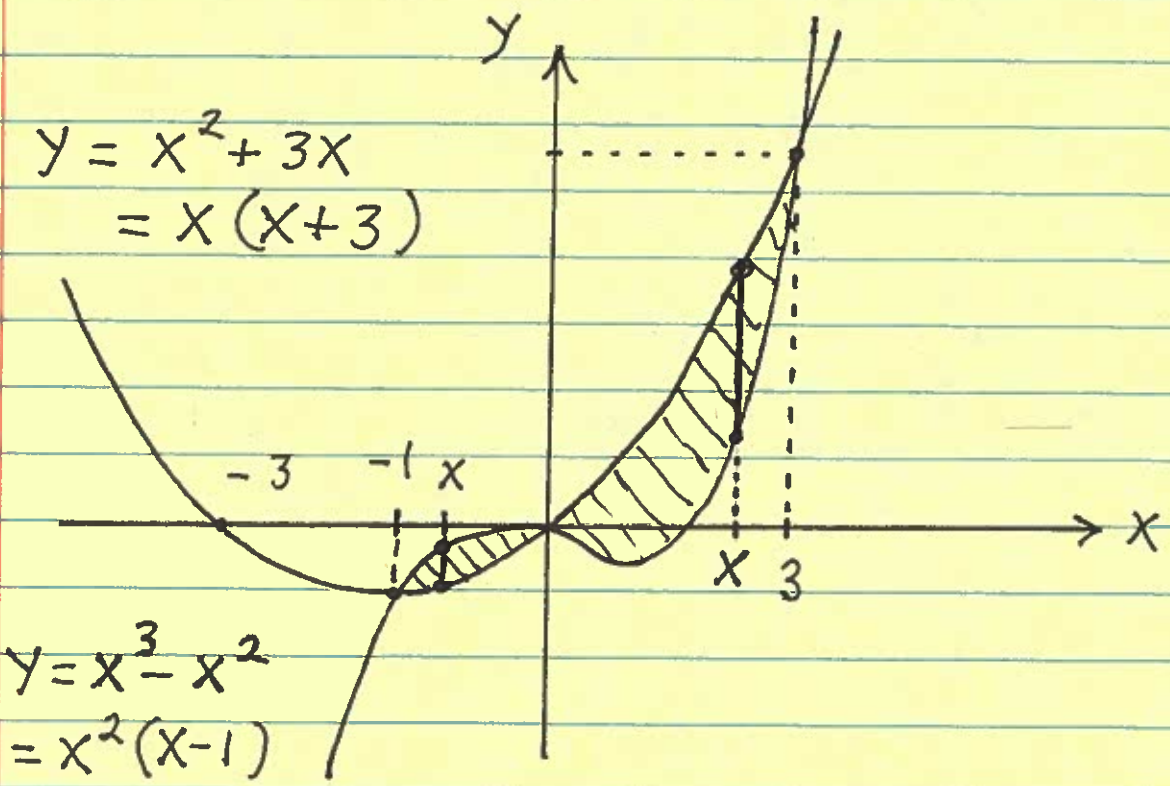
4.) $R: y = x^3 - x^2, \quad y = x^2 + 3x$

$x^3 - x^2 = x^2 + 3x \rightarrow$

$x^3 - 2x^2 - 3x = 0 \rightarrow$

$x(x^2 - 2x - 3) = 0 \rightarrow$

$x(x-3)(x+1) = 0 \rightarrow x = 0, x = 3, x = -1$



$R: \begin{cases} -1 \le x \le 0 \\ x^2 + 3x \le y \le x^3 - x^2 \end{cases}$ and

$\begin{cases} 0 \le x \le 3 \\ x^3 - x^2 \le y \le x^2 + 3x \end{cases}$, so

$$\text{AREA} = \iint_R 1 \, dA$$

$$= \int_{-1}^{0} \int_{x^2+3x}^{x^3-x^2} 1 \, dy \, dx$$

$$+ \int_{0}^{3} \int_{x^3-x^2}^{x^2+3x} 1 \, dy \, dx$$