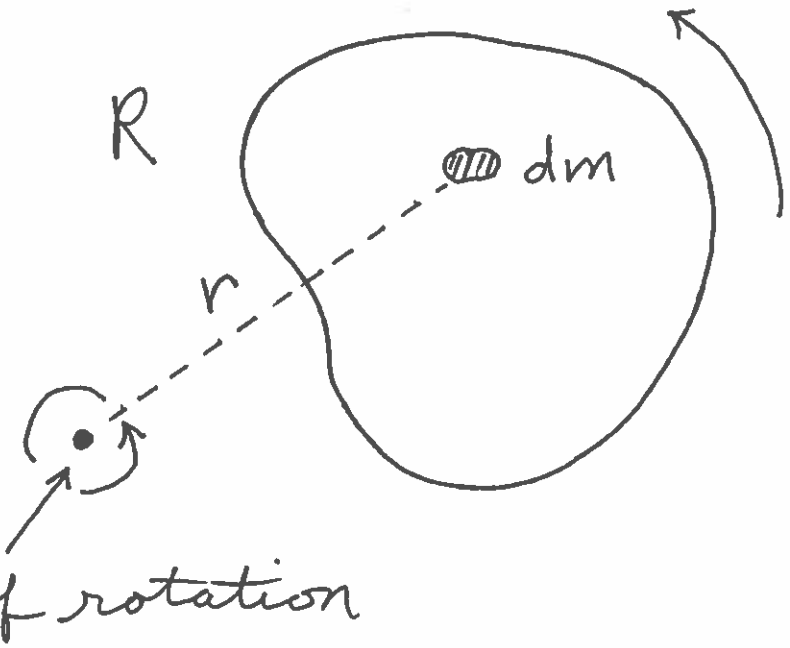
Where does the formula for Moment of Inertia come from?

Consider the TOP VIEW of a flat plate in region R rotating about an axis at an angular speed of

ω radians/sec.



assume that the density at point P is $\delta(P)$ grams/cm.^{2}, and

consider a small piece of the plate of mass dm grams at a distance of r cm. from the axis of rotation. Its velocity is

$$\left(\omega \frac{\text{rad.}}{\text{sec.}}\right)\left(\frac{2\pi r \text{ cm.}}{2\pi \text{ rad.}}\right) = \omega r \text{ cm./sec.}$$

Now the Kinetic Energy of the small piece is

$$\begin{aligned} \text{K.E.} &= \tfrac{1}{2}(\text{mass})(\text{velocity})^2 \\ &= \tfrac{1}{2}(dm)(\omega r)^2 \\ &= \tfrac{1}{2}\omega^2 \cdot r^2\, dm \\ &= \tfrac{1}{2}\omega^2 \cdot r^2 \cdot \delta(P)\, dA, \end{aligned}$$

so that the Total Kinetic Energy

of the plate is

$$K.E. = \iint_R \frac{1}{2}\omega^2 r^2 \delta(P)\, dA$$

$$= \frac{1}{2}\omega^2 \iint_R r^2 \delta(P)\, dA .$$

We define the Moment of Inertia to be :

$$M.\text{ of }I. = \iint_R r^2 \delta(P)\, dA$$