

Math 21D  
Kouba  
Discussion Sheet 9

1.) Find the area of the following surfaces  $S$ , which are directly above the rectangular region  $R$  with vertices  $(0, 0)$ ,  $(2, 0)$ ,  $(2, 4)$ , and  $(0, 4)$  in the  $xy$ -plane.

- a.) plane  $z = 5$
- b.) plane  $z = 2y$
- c.) plane  $x + 2y + 3z = 12$

2.) Find the area of the following surfaces  $S$ , which are directly above the disc  $x^2 + y^2 \leq 9$  in the  $xy$ -plane.

- a.) top half of sphere  $x^2 + y^2 + z^2 = 64$
- b.) paraboloid  $z = x^2 + y^2 + 1$
- c.) cone  $z = \sqrt{x^2 + y^2}$

3.) Let surface  $S$  be the top half of the sphere  $x^2 + y^2 + z^2 = 4$ . Define the following function  $g$  on  $S$  :  $g(P) = g(x, y, z)$  is the square of the distance from  $P$  to the  $xy$ -plane. Compute the surface integral of  $g$  over  $S$ .

4.) Let surface  $S$  be that portion of the paraboloid  $z = x^2 + y^2 + 4$  directly above the disc  $x^2 + y^2 \leq 1$  in the  $xy$ -plane. Let function  $g(x, y, z) = \sqrt{x^2 + y^2}$ . Compute the surface integral of  $g$  over  $S$ .

5.) Let surface  $S$  be that portion of the paraboloid  $z = 4 - x^2 - y^2$  cut by the plane  $z = 0$ . Find the Flux of the vector field  $\vec{F}(x, y, z) = (y)\vec{i} + (x)\vec{j} + (z)\vec{k}$  outward through the surface  $S$ .

6.) Find the Flux of the vector field  $\vec{F}(x, y, z) = (2x)\vec{i} + (-3y)\vec{j} + (z)\vec{k}$  in the direction away from the origin and across the region  $S$  in the plane  $x + 2y + 3z = 12$ , which is directly above the triangle with vertices  $(0, 0)$ ,  $(0, 2)$ , and  $(2, 6)$  in the  $xy$ -plane.

THE FOLLOWING PROBLEM IS FOR RECREATIONAL PURPOSES ONLY.

7.) Two bicyclists are twelve miles apart. They begin riding toward each other, one pedaling at 4 mph and the other at 2 mph. At the same time a bumblebee begins flying back and forth between the riders at a constant speed of 10 mph. What is the total distance the bumblebee travels by the time the riders meet ?