Math 21D (Summer Session I 2020)
Kouba
Quiz 1

Printing and signing your name below is a verification that no other person assisted you in the completion of this Quiz.

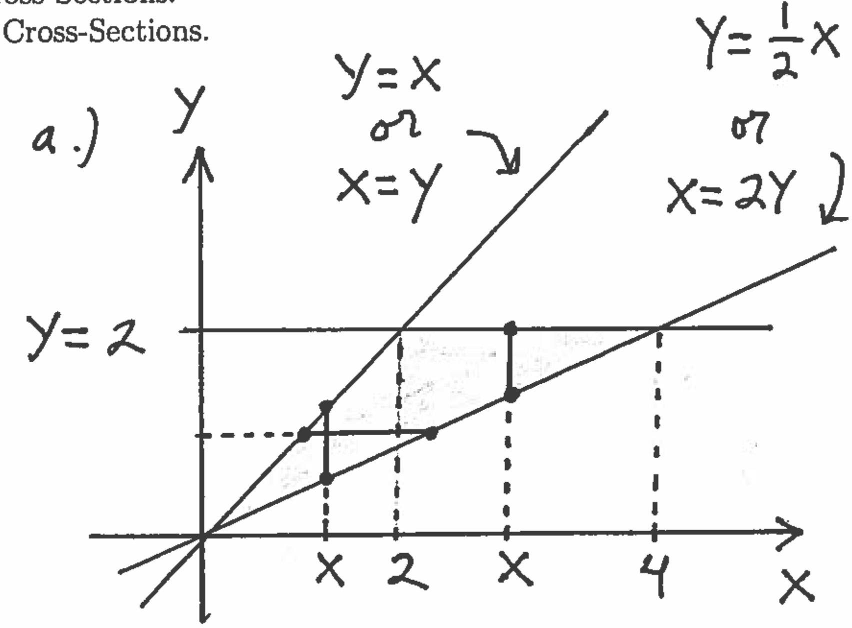
PRINT your name KEY SIGN your name ____________

Show clear, organized supporting work for your answers. Correct answers without supporting work may not receive full credit. Use of unapproved shortcuts may not receive full credit.

1.) (6 pts.) Consider region R in 2D-Space, which is bounded by the graphs of $y = x$, $y = (1/2)x$, and $y = 2$.

a.) Sketch and label region R.
b.) Describe R using Vertical Cross-Sections.
c.) Describe R using Horizontal Cross-Sections.

a.)



b.)

$$R: \begin{cases} 0 \le X \le 2 \\ \frac{1}{2}X \le Y \le X \end{cases}$$

and $$\begin{cases} 2 \le X \le 4 \\ \frac{1}{2}X \le Y \le 2 \end{cases}$$

c.) $$R: \begin{cases} 0 \le Y \le 2 \\ Y \le X \le 2Y \end{cases}$$

2.) (6 pts.) Sketch and label (in Rectangular Coordinates) the region R in 2D-Space, where R is described by

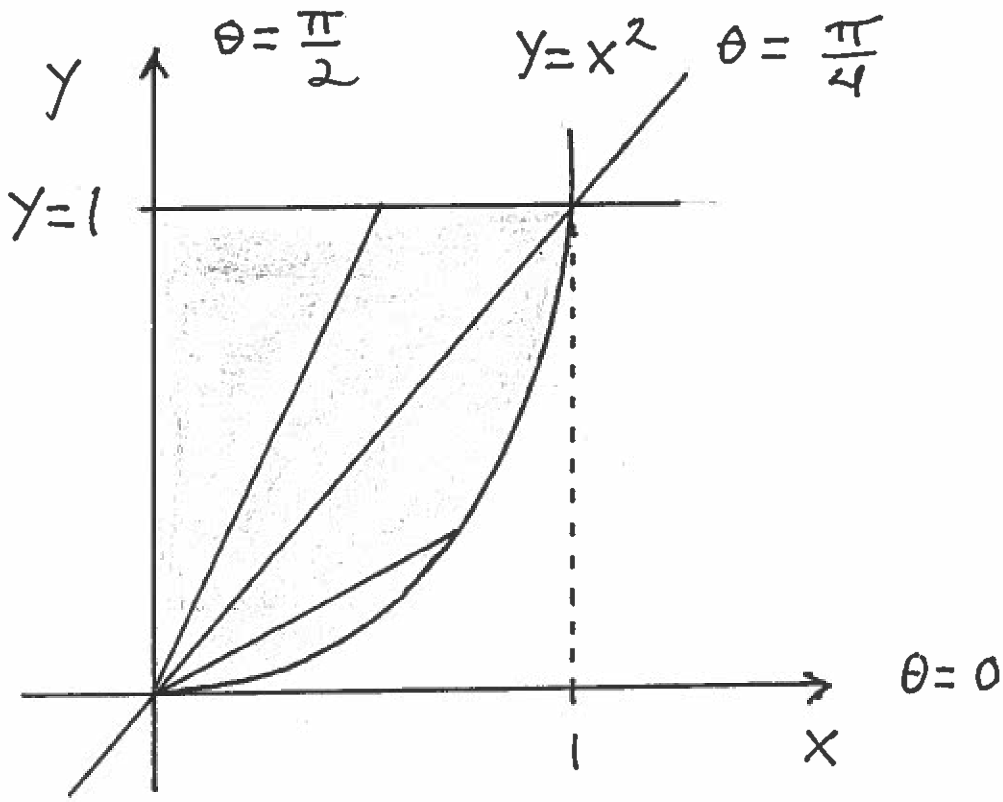
$$R: \begin{cases} 0 \le \theta \le \pi/4 \\ 0 \le r \le \sec\theta\tan\theta \end{cases} \text{ and } \begin{cases} \pi/4 \le \theta \le \pi/2 \\ 0 \le r \le \csc\theta \end{cases}$$

$$r = \sec\theta \tan\theta = \frac{1}{\cos\theta} \cdot \frac{\sin\theta}{\cos\theta} \rightarrow$$

$$r\cos^2\theta = \sin\theta \rightarrow r^2\cos^2\theta = r\sin\theta \rightarrow$$

$$(r\cos\theta)^2 = r\sin\theta \rightarrow x^2 = y \ ;$$

$$r = \csc\theta = \frac{1}{\sin\theta} \rightarrow r\sin\theta = 1 \rightarrow y = 1 \ ;$$



3.) (10 pts. each) Evaluate the following (three) Double Integrals.

a.) $\int_0^{\sqrt{\pi}} \int_0^{x^2} x \cos y \; dy\, dx$

$$= \int_0^{\sqrt{\pi}} \left(x \sin y \Big|_{y=0}^{y=x^2} \right) dx$$

$$= \int_0^{\sqrt{\pi}} x \sin(x^2) \; dx$$

$$= -\frac{1}{2} \cos(x^2) \Big|_0^{\sqrt{\pi}}$$

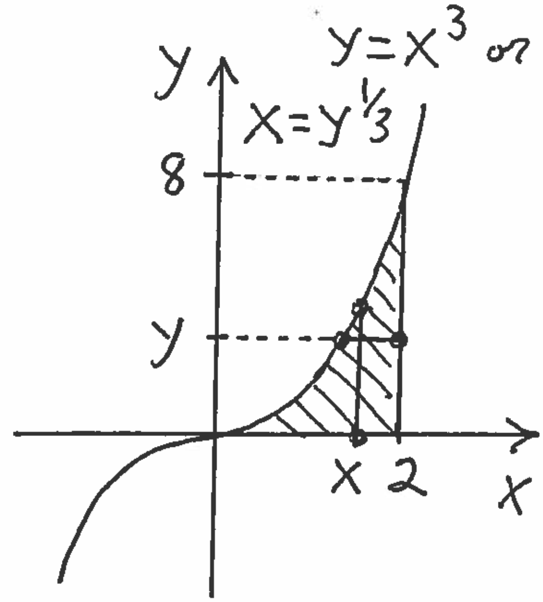
$$= \frac{-1}{2} \cos \pi - \frac{-1}{2} \cos 0$$

$$= \frac{-1}{2}(-1) + \frac{1}{2}(1)$$

$$= 1$$

b.) $\int_0^8 \int_{y^{1/3}}^2 \sqrt{1+x^4}\, dx\, dy$

(Switch ORDER of Integration)



$$= \int_0^2 \int_0^{x^3} \sqrt{1+x^4}\, dy\, dx$$

$$= \int_0^2 \sqrt{1+x^4} \cdot y \Big|_{y=0}^{y=x^3} dx$$

$$= \int_0^2 x^3 (1+x^4)^{1/2} dx$$

$$= \frac{2}{3} \cdot \frac{1}{4} (1+x^4)^{3/2} \Big|_0^2$$

$$= \frac{1}{6}(17)^{3/2} - \frac{1}{6}$$

$$R: \begin{cases} 0 \le y \le 8 \\ y^{1/3} \le x \le 2 \end{cases}$$

OR

$$R: \begin{cases} 0 \le x \le 2 \\ 0 \le y \le x^3 \end{cases}$$

c.) $\displaystyle\int_{-3}^{3}\int_{0}^{\sqrt{9-x^2}} \sqrt{x^2+y^2}\; dy\, dx$ (HINT: Switch to Polar Coordinates.)

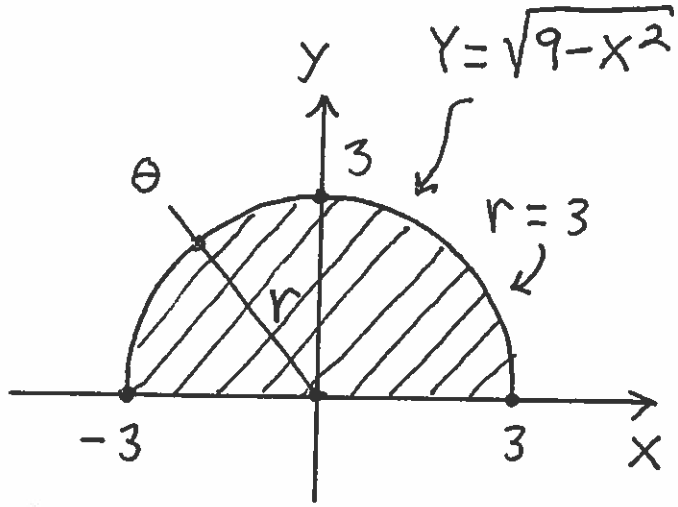
$$= \int_0^{\pi}\int_0^{3} \sqrt{r^2}\cdot r \, dr\, d\theta$$

$$= \int_0^{\pi}\int_0^{3} r^2 \, dr\, d\theta$$

$$= \int_0^{\pi} \left(\frac{1}{3} r^3 \Big|_0^3 \right) d\theta$$

$$= \int_0^{\pi} 9 \, d\theta = 9\theta \Big|_0^{\pi}$$

$$= 9\pi$$

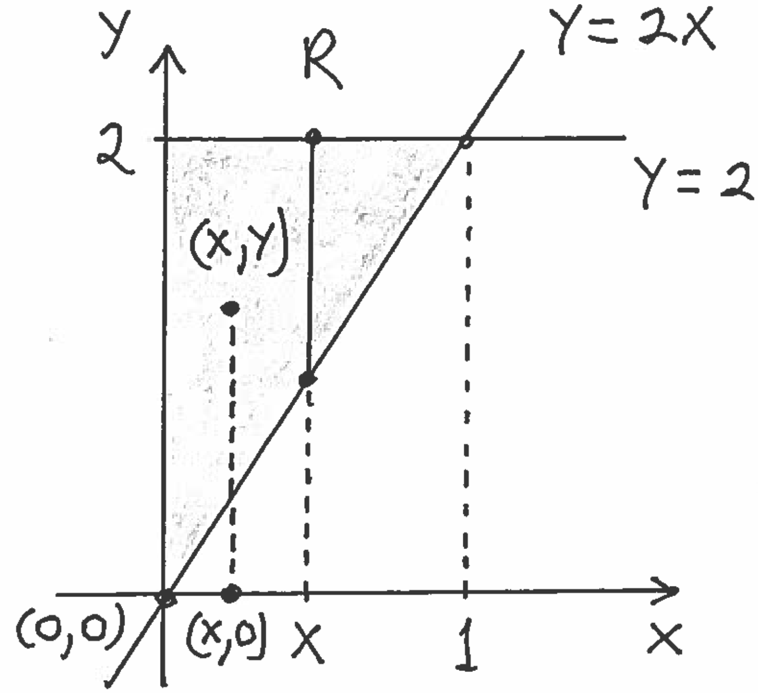


$$R: \begin{cases} -3 \le X \le 3 \\ 0 \le Y \le \sqrt{9-X^2} \end{cases}$$

OR

$$R: \begin{cases} 0 \le \theta \le \pi \\ 0 \le r \le 3 \end{cases}$$

4.) (10 pts.) Consider region R in 2D-Space, which is bounded by the graphs of $y = 2x$, $y = 2$, and $x = 0$. Find the Average Distance (SET UP ONLY) from points (x, y) in R to the $x-$axis.



Area of R

$$= \int_0^1 \int_{2x}^2 1 \, dy \, dx \; ;$$

distance from (x, y) to x-axis is distance $= y$;

$$R: \begin{cases} 0 \le x \le 1 \\ 2x \le y \le 2 \end{cases}$$

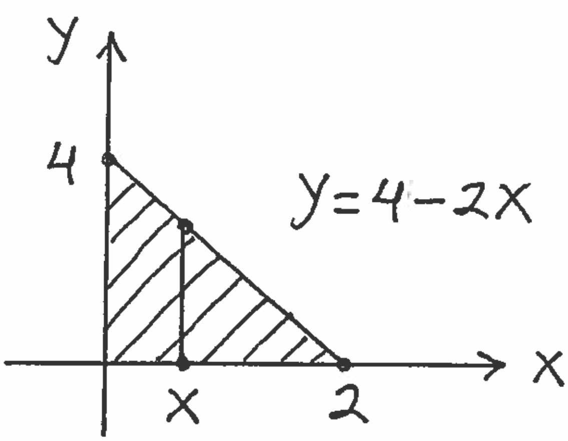
Averge distance

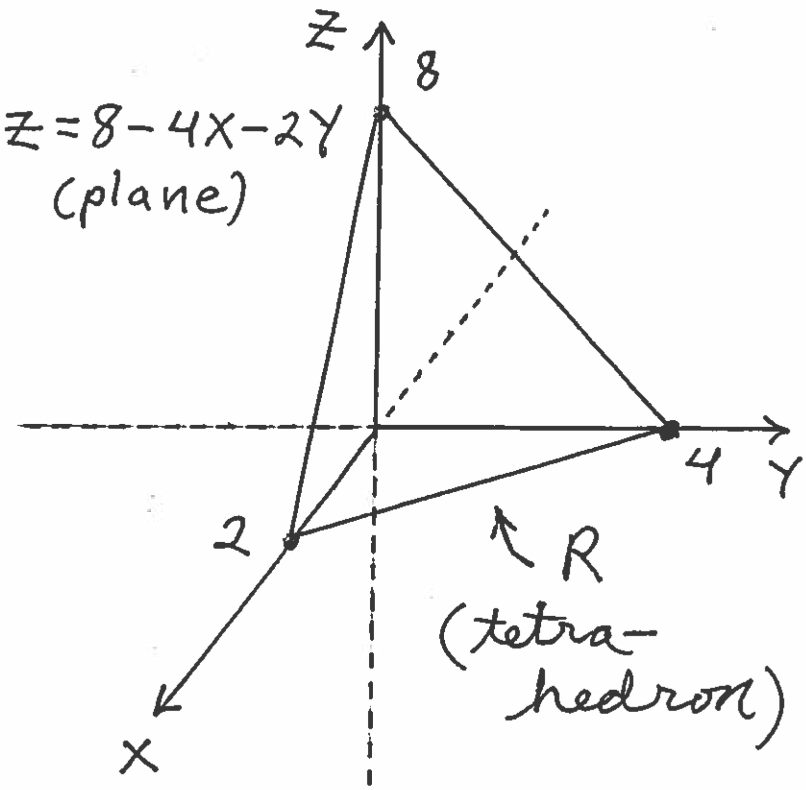
$$= \frac{1}{\text{Area of } R} \int_0^1 \int_{2x}^2 y \, dy \, dx$$

5.) (8 pts.) Sketch and label the solid S in 3D-Space whose volume is given by the following Double Integral.

$$\int_0^2 \int_0^{4-2x} (8 - 4x - 2y)\ dy\, dx$$

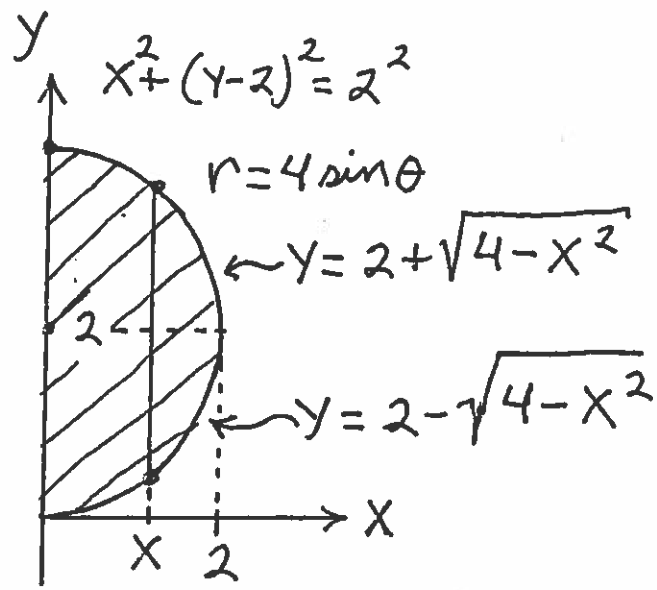
$$R: \begin{cases} 0 \le x \le 2 \\ 0 \le y \le 4 - 2x \end{cases}$$





6.) (10 pts.) Consider region R in 2D-Space, which is bounded by the y-axis and the right half of the circle given in polar coordinates by $r = 4\sin\theta$. Find the x-coordinate of the Centroid of R (SET UP ONLY) using Rectangular Coordinates.

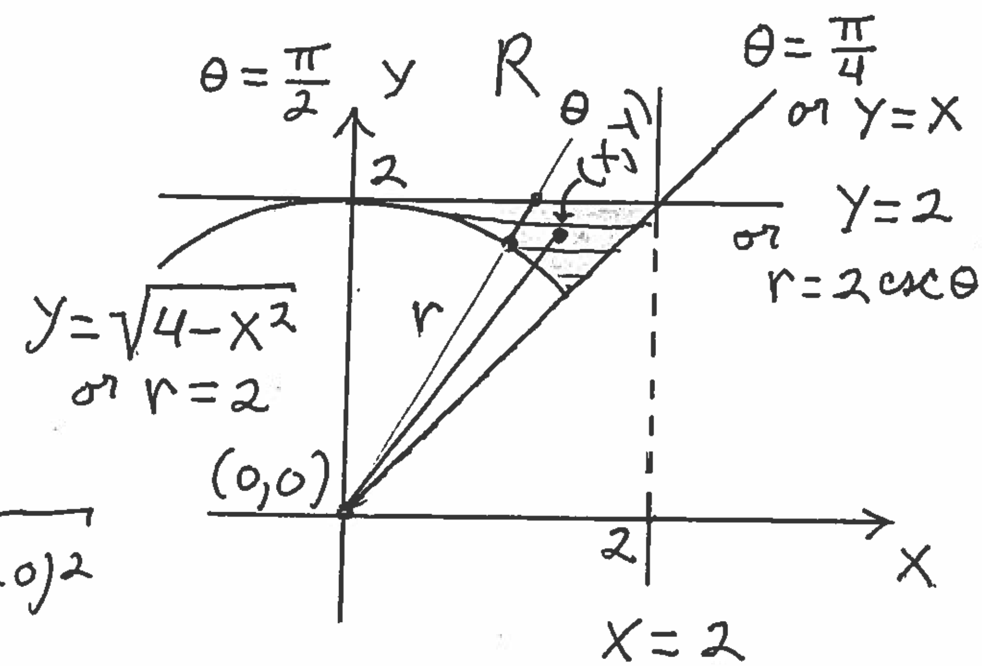
$$\bar{x} = \frac{\displaystyle\int_0^2 \int_{2-\sqrt{4-x^2}}^{2+\sqrt{4-x^2}} x \, dy\, dx}{\displaystyle\int_0^2 \int_{2-\sqrt{4-x^2}}^{2+\sqrt{4-x^2}} 1 \, dy\, dx}$$



$$R: \begin{cases} 0 \le x \le 2 \\ 2-\sqrt{4-x^2} \le y \le 2+\sqrt{4-x^2} \end{cases}$$

7.) (12 pts.) Consider the flat plate in region R in 2D-Space, which is bounded by the graphs of $y = \sqrt{4-x^2}$, $y = 2$, and $y = x$ in the first quadrant. Assume that the density at point $P = (x, y)$ in R is given by $\delta(P) = \delta(x, y) = x^2 y \;\; gm/cm^2$. Find the Moment of Inertia of R about the origin (SET UP ONLY) using Polar Coordinates.

Distance from $(0,0)$ to (x,y) is



$$L = \sqrt{(x-0)^2 + (y-0)^2}$$

$$= \sqrt{x^2 + y^2}$$

$$= \sqrt{r^2} = r$$

$$R: \begin{cases} \frac{\pi}{4} \le \theta \le \frac{\pi}{2} \\ 2 \le r \le 2\csc\theta \end{cases},$$

$$\text{M. of I.} = \iint_R (\text{distance})^2 \, \delta(P) \, dA$$

$$= \int_{\pi/4}^{\pi/2} \int_{2}^{2\csc\theta} (r)^2 (r\cos\theta)^2 (r\sin\theta) \cdot r \, dr \, d\theta$$

$(gm.)(cm.^2)$

8.) (8 pts.) Evaluate the following triple integral.

$$\int_0^1 \int_0^y \int_0^{x^2} (2xy + 6z)\ dz\, dx\, dy$$

$$= \int_0^1 \int_0^Y (2XYZ + 3Z^2) \Big|_{Z=0}^{Z=X^2} dx\, dy$$

$$= \int_0^1 \int_0^Y (2XY(X^2) + 3(X^2)^2)\, dx\, dy$$

$$= \int_0^1 \int_0^Y (2X^3Y + 3X^4)\, dx\, dy$$

$$= \int_0^1 \left(\tfrac{1}{2}X^4Y + \tfrac{3}{5}X^5\right) \Big|_{X=0}^{X=Y} dy$$

$$= \int_0^1 \left(\tfrac{1}{2}Y^5 + \tfrac{3}{5}Y^5\right) dy$$

$$= \int_0^1 \left(\tfrac{5}{10}Y^5 + \tfrac{6}{10}Y^5\right) dy$$

$$= \int_0^1 \tfrac{11}{10} Y^5\, dy$$

$$= \tfrac{11}{10} \cdot \tfrac{1}{6} Y^6 \Big|_0^1$$

$$= \tfrac{11}{60}$$

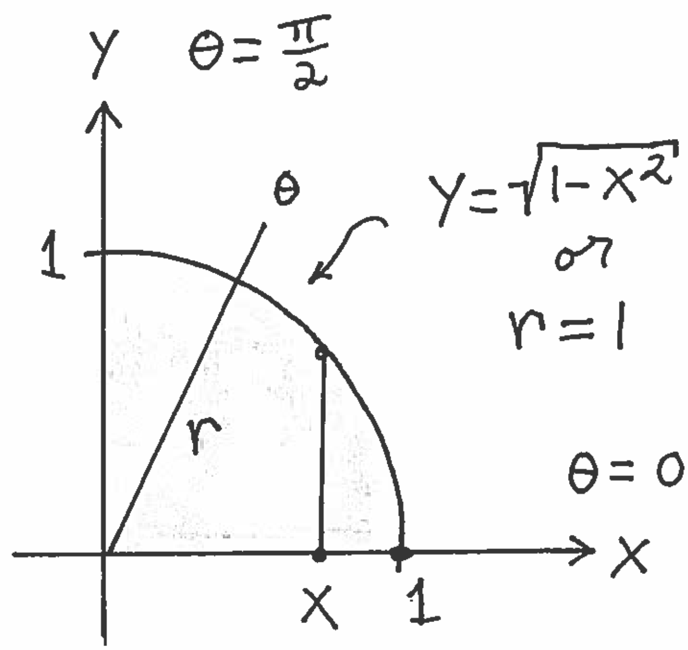
9.) (10 pts.) Convert the following Triple Integral to Cylindrical Coordinates and then evaluate it.

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{(x^2+y^2+1)^{1/4}} 2z \; dz\, dy\, dx$$

$$R: \begin{cases} 0 \le x \le 1 \\ 0 \le y \le \sqrt{1-x^2} \end{cases}$$

or

$$R: \begin{cases} 0 \le \theta \le \frac{\pi}{2} \\ 0 \le r \le 1 \end{cases}$$



$$z = (x^2+y^2+1)^{1/4} = (r^2+1)^{1/4} \;;$$

$$= \int_0^{\frac{\pi}{2}} \int_0^1 \int_0^{(r^2+1)^{1/4}} 2z \cdot r \; dz\, dr\, d\theta$$

$$= \int_0^{\frac{\pi}{2}} \int_0^1 \left(z^2 r \Big|_{z=0}^{(r^2+1)^{1/4}} \right) dr\, d\theta$$

$$= \int_0^{\frac{\pi}{2}} \int_0^1 r(r^2+1)^{1/2} \, dr\, d\theta$$

$$= \int_0^{\frac{\pi}{2}} \left(\frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{3} (r^2+1)^{3/2} \Big|_{r=0}^{r=1} \right) d\theta$$

$$= \int_0^{\frac{\pi}{2}} \left(\frac{1}{3}(2)^{3/2} - \frac{1}{3} \right) d\theta = \left(\frac{1}{3}(2)^{3/2} - \frac{1}{3} \right) \theta \Big|_0^{\frac{\pi}{2}}$$

$$= \left(\frac{1}{3}(2)^{3/2} - \frac{1}{3} \right) \frac{\pi}{2}$$