Math 21D
Kouba
Applications of Double Integrals

Let R be a flat region in two-dimensional space and let $\delta(P)$ be the density of the region at point $P = (x, y)$.

1.) AREA : $\int_R 1\,dA$ represents the *area* of region R.

2.) AVERAGE VALUE : $\dfrac{1}{Area\ of\ R}\int_R f(x,y)\,dA$ represents the *average value* of function $z = f(x, y)$ over region R.

3.) MASS : $\int_R \delta(P)\,dA$ represents the *mass* of region R.

4.) VOLUME : $\int_R f(P)\,dA$ represents the *volume* of the solid region defined on region R with height $f(P)$ at point P.

5.) MOMENT :

a.) $\int_R (x-a)\delta(P)\,dA$ represents the *moment* of region R about the vertical line $x = a$.

b.) $\int_R (y-b)\delta(P)\,dA$ represents the *moment* of region R about the horizontal line $y = b$.

6.) CENTER OF MASS, $(\bar{x}, \bar{y})$:

a.) $\bar{x} = \dfrac{\int_R x\delta(P)\,dA}{\int_R \delta(P)\,dA}$ represents the *x-coordinate* of the center of mass of region R.

b.) $\bar{y} = \dfrac{\int_R y\delta(P)\,dA}{\int_R \delta(P)\,dA}$ represents the *y-coordinate* of the center of mass of region R.

7.) CENTROID, $(\bar{x}, \bar{y})$:

a.) $\bar{x} = \dfrac{\int_R x\,dA}{\int_R 1\,dA}$ represents the *x-coordinate* of the centroid of region R.

b.) $\bar{y} = \dfrac{\int_R y\,dA}{\int_R 1\,dA}$ represents the *y-coordinate* of the centroid of region R.

NOTE : The formulas for centroid follow immediately from the formulas for center of mass by letting density $\delta(P) = 1$.

8.) MOMENT OF INERTIA : $\int_R (distance)^2\delta(P)\,dA$ represents the *moment of inertia* of region R, where *distance* refers to the distance from point $P = (x, y)$ in region R to either a point or axis (line) of rotation.