Applications of Surface Integrals:

Let $\mathcal{S}$ be a surface and assume the density at point $P=(x,y,z)$ on $\mathcal{S}$ is $\delta(P)$ (mass/area units).

1.) Area $\mathcal{S} = \iint_{\mathcal{S}} 1 \, dS$

2.) Mass $\mathcal{S} = \iint_{\mathcal{S}} \delta(P) \, dS$

3.) Moments about planes:

a.) $M_{x=a} = \iint_{\mathcal{S}} (x-a)\,\delta(P) \, dS$

b.) $M_{y=a} = \iint_{\mathcal{S}} (y-a)\,\delta(P) \, dS$

c.) $M_{z=a} = \iint_{\mathcal{S}} (z-a)\,\delta(P) \, dS$

4.) Center of Mass $(\bar{x}, \bar{y}, \bar{z})$:

$$\bar{x} = \frac{\iint_{\mathcal{S}} x\,\delta(P)\,dS}{\iint_{\mathcal{S}} \delta(P)\,dS}, \quad \bar{y} = \frac{\iint_{\mathcal{S}} y\,\delta(P)\,dS}{\iint_{\mathcal{S}} \delta(P)\,dS}, \quad \bar{z} = \frac{\iint_{\mathcal{S}} z\,\delta(P)\,dS}{\iint_{\mathcal{S}} \delta(P)\,dS}$$

5.) Centroid $(\bar{x}, \bar{y}, \bar{z})$:

$$\bar{x} = \frac{\iint_{\mathcal{S}} x\,dS}{\iint_{\mathcal{S}} 1\,dS}, \quad \bar{y} = \frac{\iint_{\mathcal{S}} y\,dS}{\iint_{\mathcal{S}} 1\,dS}, \quad \bar{z} = \frac{\iint_{\mathcal{S}} z\,dS}{\iint_{\mathcal{S}} 1\,dS}$$

6.) Moment of Inertia:

$$\text{M. of I.} = \iint_{\mathcal{L}} (\text{distance})^2 \, \delta(P) \, dS,$$

where distance refers to the distance from point $P = (x, y, z)$ on surface $\mathcal{L}$ to an axis of rotation or a plane.