Math 22A
Kouba
Determinants

DEFINITION: Let $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ be a 2×2 matrix. The **determinant** of A is

$$det(A) = a_{11}a_{22} - a_{12}a_{21}$$

DEFINITION: Let $A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ & & & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}$ be a $n \times n$ matrix.

1.) The **minor** of entry a_{ij}, written M_{ij}, is the determinant of the matrix remaining after the ith row and jth column of matrix A are deleted.

2.) The **cofactor** of entry a_{ij}, written C_{ij}, is the signed minor

$$C_{ij} = (-1)^{i+j} M_{ij}$$

3.) The **determinant** of A is

a.) $det(A) = a_{i1}C_{i1} + a_{i2}C_{i2} + \cdots + a_{in}C_{in}$ (ith row expansion)

OR

b.) $det(A) = a_{1j}C_{1j} + a_{2j}C_{2j} + \cdots + a_{nj}C_{nj}$ (jth column expansion)

QUESTION: Why do 3.)a.) and 3.)b.) give the same number ? (See link on my webpage for details.)

TRUE or FALSE: If A and B are 2×2 matrices, then $det(A + B) = det(A) + det(B)$.

TRUE or FALSE: If A is a 2×2 matrix, then $det(kA) = k^2\ det(A)$.