Math 22A
Kouba
Vectors in $\mathbb{R}^2$, $\mathbb{R}^3$, and $\mathbb{R}^n$

DEFINITION 3 If $\mathbf{v} = \overrightarrow{(v_1, v_2, \ldots, v_n)}$ and $\mathbf{w} = \overrightarrow{(w_1, w_2, \ldots, w_n)}$ are vectors in R^n, and if k is any scalar, then we define

$$\mathbf{v} + \mathbf{w} = \overrightarrow{(v_1 + w_1, v_2 + w_2, \ldots, v_n + w_n)} \tag{10}$$

$$k\mathbf{v} = \overrightarrow{(kv_1, kv_2, \ldots, kv_n)} \tag{11}$$

$$-\mathbf{v} = \overrightarrow{(-v_1, -v_2, \ldots, -v_n)} \tag{12}$$

$$\mathbf{w} - \mathbf{v} = \mathbf{w} + (-\mathbf{v}) = \overrightarrow{(w_1 - v_1, w_2 - v_2, \ldots, w_n - v_n)} \tag{13}$$

THEOREM 3.1.1 *If* $\mathbf{u}$, $\mathbf{v}$, *and* $\mathbf{w}$ *are vectors in* R^n, *and if* k *and* m *are scalars, then:*

(*a*) $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$

(*b*) $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$

(*c*) $\mathbf{u} + \mathbf{0} = \mathbf{0} + \mathbf{u} = \mathbf{u}$

(*d*) $\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$

(*e*) $k(\mathbf{u} + \mathbf{v}) = k\mathbf{u} + k\mathbf{v}$

(*f*) $(k + m)\mathbf{u} = k\mathbf{u} + m\mathbf{u}$

(*g*) $k(m\mathbf{u}) = (km)\mathbf{u}$

(*h*) $1\mathbf{u} = \mathbf{u}$

THEOREM 3.1.2 *If* $\mathbf{v}$ *is a vector in* R^n *and* k *is a scalar, then:*

(*a*) $0\mathbf{v} = \mathbf{0}$

(*b*) $k\mathbf{0} = \mathbf{0}$

(*c*) $(-1)\mathbf{v} = -\mathbf{v}$