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Analysis of Higher Order Moore-Greitzer Compressor Models *

J. Sean Humbert
Division of Engineering and Applied Science
California Institute of Technology
Pasadena, CA 91125
jshumber@indra.caltech.edu

Arthur J. Krener
Department of Mathematics
University of California
Davis, CA 95616
ajkrener@ucdavis.edu

Abstract

In this paper we investigate the behavior of higher order Galerkin expansions of the Moore-Greitzer model of general transients in aeroengine compression systems. We assume steady state entrainment of the higher Fourier modes of the rotating stall cell which establishes a framework for a simplified numerical analysis of the bifurcating solutions corresponding to rotating stall. For small values of the Greitzer surge parameter (B) we discuss general trends in the character of the pure stall solutions. The rotating stall characteristic is shown to exhibit deep hysteresis with a cubic compressor characteristic, establishing the fact that deep hysteresis to a certain extent is a multi-mode phenomena. Elimination of the hysteresis associated with the bifurcation into stall is accomplished in simulations with a combined feedback on the displacement from the peak of the compressor characteristic and the magnitude of the first mode amplitude of the stall cell. Behavior for larger values of the B parameter is also investigated and novel surge/stall relaxation oscillations corresponding to classic surge are discovered.

1. Introduction

Rotating stall and surge are fluid dynamic instabilities which limit performance of axial compression systems. Deep surge is a time dependent axisymmetric flow characterized by oscillations in both pressure rise and mean flow. Pure rotating stall, on the other hand, is a steady (in the rotating frame), non-axisymmetric flow whose frequency is typically an order of magnitude larger than that of surge. The case where these two instabilities coexist is termed classic surge. Recovery from these conditions is characterized by hysteresis, where the throttle must be significantly opened past the point of inception in order to recover steady axisymmetric flow. The Moore-Greitzer equations are an attempt to model the nonlinear phenomena of rotating stall and surge in these systems using first principles. The derivation of the complete set of equations can be found in [1].

Desired operating points of this system are those that provide the largest pressure rise across the compressor. However, increased susceptibility to surge and stall precludes operation at these setpoints. Reduced order ODE models of stall and surge phenomena provide a framework to assess the performance of various state feedbacks designed to minimize the effects of rotating stall and surge. In previous work, Moore and Greitzer [1] developed a single spatial harmonic expansion of the resulting equations. The bifurcations in this model were studied in depth by McCaughan [2]. In many papers, too numerous to mention here, feedbacks were shown to eliminate the hysteresis associated with the bifurcation into stall of the single mode expansion. It was also shown in [3] that various feedbacks which eliminate hysteresis in the single mode expansion fail to do so when an additional spatial harmonic is included in the model.

In this paper we investigate the dynamics of multi-mode Galerkin expansions of the Moore-Greitzer equations. In Section 2 we introduce phase entrainment states which establish a framework for simple numerical analysis of non-axisymmetric periodic solutions as well as analysis of the performance

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of feedbacks in the multi-mode environment. In Section 3 we present a detailed numerical analysis of the three mode model ($MG8$) and note the general trends seen when more Fourier modes are included in the dynamics. Finally, in Section 4 we introduce a feedback which eliminates the hysteresis associated with the bifurcation into stall for the models we have simulated.

1.1. Complete Moore-Greitzer Model

The complete model, whose derivation can be found in [1], is as follows

$$l_c \frac{d\Psi}{d\xi} = \frac{1}{4B^2}(\Phi - \Phi_T) \quad (1)$$

$$\Psi(\xi) = \Psi_c(\Phi + (\tilde{\phi}'_\eta)_0) - l_c\frac{d\Phi}{d\xi} - m(\tilde{\phi}'_\xi)_0 - \frac{\mu}{2}(2\tilde{\phi}'_{\xi\eta} + \tilde{\phi}'_{\eta\theta})_0 \quad (2)$$

$$l_c\frac{d\Phi}{d\xi} = -\Psi(\xi) + \frac{1}{2\pi}\int_0^{2\pi}\Psi_c(\Phi + (\tilde{\phi}'_\eta)_0)d\theta \quad (3)$$

The dependent variables of this system are the annulus averaged pressure rise coefficient $\Psi(\xi)$, the annulus averaged axial flow coefficient $\Phi(\xi)$, and the upstream disturbance potential $\tilde{\phi}'(\eta, \theta, \xi)$, whose axial and circumferential partials give the local flow disturbance in the axial and circumferential directions. Independent variables include the time in wheel radians ξ, the circumferential coordinate θ, and the axial coordinate η. Equation (1) is an ODE in ξ which results from a mass balance of the plenum, (2) is a PDE in ξ and θ from the momentum balance of the system evaluated at the compressor face ($\eta = 0$), and (3) is an ODE in ξ which results from averaging out the circumferential dependence in (2). Note the subscript 0 denotes evaluation at $\eta = 0$, and the subscripts ξ, θ, η denote partial differentiation.

The compressor characteristic $\Psi_c(\Phi)$ is the response of the compressor for steady axisymmetric flow. For our analysis we will use the general Moore-Greitzer cubic from [1]:

$$\Psi_c(\Phi) = \Psi_0 + h\left[1 + \frac{3}{2}\left(\frac{\Phi}{w} - 1\right) - \frac{1}{2}\left(\frac{\Phi}{w} - 1\right)^3\right] \quad (4)$$

The throttle characteristic $\Phi_T(\Psi)$ represents the pressure loss across the throttle, assumed parabolic in Φ:

$$\Phi_T = (K_T + u)\sqrt{\Psi} \quad (5)$$

The variable u represents a feedback control on the operating point of the system. Parameters of the model define compressor geometry and operation characteristics. In our analysis we will focus on two parameters, K_T and B. The throttle coefficient K_T adjusts the position of the throttle, hence the operating point of the system. B is the Greitzer stability parameter which determines whether a given compressor is more likely to enter surge or rotating stall. Additional parameters are defined in [1]. We will refer to the collection of parameters as

$$p = [K_T\ \Psi_0\ h\ w\ m.l_c\ \mu\ B]' \quad (6)$$

1.2. Galerkin Projections of the Complete Model

The procedure for developing finite dimensional projections of the full Moore-Greitzer model is outlined in [4]. Essentially a system of ODEs that approximate the non-axisymmetric dynamics of (2) is constructed by representing the the disturbance potential $\tilde{\phi}'$ with a finite number of Fourier modes

$$\tilde{\phi}'(\xi, \theta, \eta) = \sum_{k=1}^{n}\frac{1}{k}\exp(k\eta)(A_k(\xi)\cos(k\theta - \Theta_k(\xi))) \quad (7)$$

and projecting the PDE onto the subspace spanned by this basis. The resulting systems have an independent variable of nondimensional time ξ measured in rotor radians and dependent variables $A_k(\xi)$ and $\Theta_k(\xi)$, the amplitudes and phases of each spatial Fourier mode included in the truncation, respectively. The ODEs ($k = 1, \cdots, n$) are

$$\dot{A}_k = \frac{\alpha_k}{\pi}\left[\int_0^{2\pi}\Psi_c(\Phi + (\tilde{\phi}'_\eta)_0)\cos(k\theta - \Theta_k)d\theta\right]$$

$$\dot{\Theta}_k = \alpha_k\left[-\frac{\mu k}{2} + \frac{1}{\pi A_k}\int_0^{2\pi}\Psi_c(\Phi + (\tilde{\phi}'_\eta)_0)\sin(k\theta - \Theta_k)d\theta\right] \quad (8)$$

where

$$\alpha_k = \frac{k}{m + k\mu} \quad (9)$$

The dynamics in expanded form for various truncations of the disturbance potential appears in [4], hence omitted in this treatment.

2. General Multi-Mode Remarks

These systems of dimension $2n + 2$ are composed of two ODEs (1),(3) describing the time evolution of the surge variables Φ and Ψ, and $2n$ ODEs (8) for the time evolution of the amplitude A_k and phase Θ_k of the k^{th} spatial Fourier mode of the stall cell. These systems can be viewed as $n+1$ coupled oscillators with the following form:

$$\dot{x} = f(x,p), \quad x = \begin{bmatrix} \Phi(\xi) \\ \Psi(\xi) \\ A_1(\xi) \\ \Theta_1(\xi) \\ \vdots \\ A_n(\xi) \\ \Theta_n(\xi) \end{bmatrix} \quad p = \begin{bmatrix} K_T \\ \Psi_0 \\ h \\ w \\ m \\ l_c \\ \mu \\ B \end{bmatrix} \tag{10}$$

where x is the state vector of the system, p is the parameter vector, and f is a nonlinear function. We will begin our analysis of these higher order expansions by assuming a small B parameter and concentrating on the non-axisymmetric dynamics defined by (10). For large values of the throttle coefficient there is a stable axisymmetric equilibrium defined by the intersections of the compressor characteristic (4) and the throttle characteristic (5). As the system is throttled down through the peak the axisymmetric equilibrium loses linear stability via a *multiple Hopf* bifurcation, as a pair of complex eigenvalues for each spatial mode of rotating stall cross the imaginary axis and become unstable. Multiple periodic solutions bifurcate from this point, however there is only one that becomes stable for the range of throttle settings. For our analysis we will assume that the phases of the various Fourier modes of this stable periodic solution become entrained at steady state, which reduces the problem of characterizing the non-axisymmetric equilibria to that of numerical continuation. We will also use this framework to investigate the dynamics of the system for larger B parameters, as well as demonstrate a feedback which eliminates the hysteresis associated with the bifurcation into rotating stall.

2.1. Entrainment of Higher Modes

Extensive simulations of (10) have shown that at steady state the solutions (traveling waves) corresponding to rotating stall maintain a fixed shape. We will refer to this phenomena as *entrainment*, or phase locking of the modes. In this situation, for a given mode, its amplitude and the difference between its phase and the product of its harmonic number and the phase of the first mode would be constant:

$$\begin{aligned} A_k &= constant \\ \Theta_k - k\Theta_1 &= constant \end{aligned} \tag{11}$$

This becomes clear when we rewrite the contribution from each spatial mode of the truncated Fourier series for the stall cell using (11):

$$A_k cos(k\theta - \Theta_k) = A_k cos(k(\theta - \Theta_1) - constant) \tag{12}$$

Upon examination of the entire series $(k = 1, \cdots, n)$, we find during entrainment Θ_1 is the only variable of the series which is a function of time. This is equivalent to the solutions (stall cells) rotating around the annulus with fixed shape.

Additionally we point out that in the dynamics of the multi-mode expansions we have investigated the phases always appear in combinations that are harmonically balanced, where the weighted sum of the harmonic orders is zero. It is simple to show, hence omitted, that any phase combination with this property can be rewritten as a linear combination of the following phase entrainment states:

$$\zeta_i = \Theta_{i+1} - (i+1)\Theta_1 \quad , \quad i = 1, \cdots, n-1 \tag{13}$$

The dynamics for each of these new states is then

$$\dot{\zeta}_i = \dot{\Theta}_{i+1} - (i+1)\dot{\Theta}_1 \quad , \quad i = 1, \cdots, n-1 \tag{14}$$

Applying this change of coordinates to (10) we obtain systems with the following forms:

$$\dot{z} = g(z,p), \quad z = \begin{bmatrix} \Phi(\xi) \\ \Psi(\xi) \\ A_1(\xi) \\ \vdots \\ A_n(\xi) \\ \zeta_1(\xi) \\ \vdots \\ \zeta_{n-1}(\xi) \end{bmatrix} \quad p = \begin{bmatrix} K_T \\ \Psi_0 \\ h \\ w \\ m \\ l_c \\ \mu \\ B \end{bmatrix} \tag{15}$$

The fixed points of the system (15) are solutions of $g(z,p) = 0$. The polar coordinate representation of the dynamics is singular in the axisymmetric case $(A_k = 0, k = 1, \cdots, n)$, hence we will focus on the character of the non-axisymmetric solutions for the range of throttle coefficients K_T. We will refer to these solutions as the entrained equilibria of the system:

$$g(z, K_T) = 0 \tag{16}$$

Intersections of the throttle characteristic (5) with these static equilibria (16) correspond to rotating stall limit cycles where the flow coefficient Φ, the pressure rise coefficient Ψ, the amplitudes of each spatial mode of the stall cell A_k, and the phase entrainment states ζ_i are all *constant*. Recall this corresponds to a stall cell (traveling wave) rotating around the annulus with fixed shape.

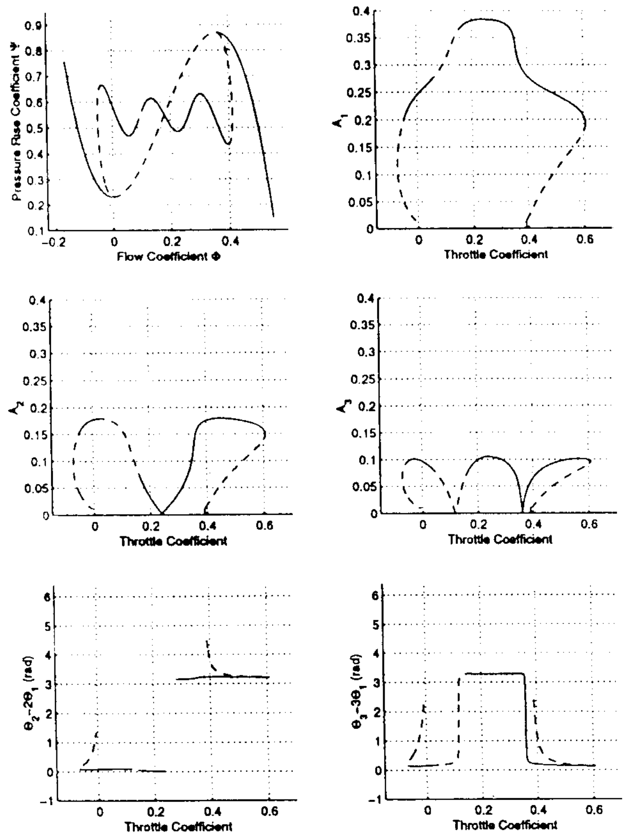

Figure 1: Primary Stall Equilibria - $MG8$

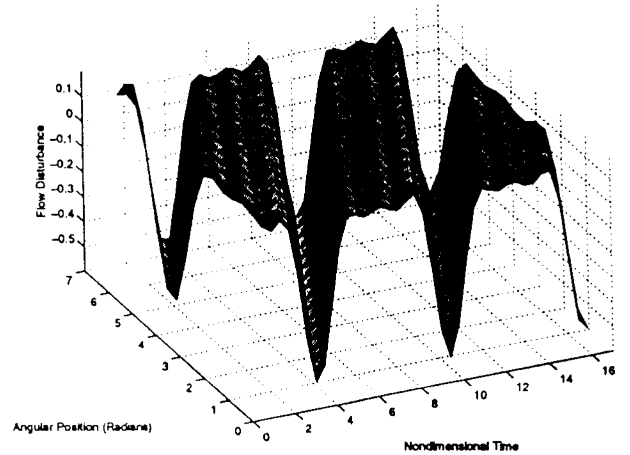

Figure 2: Stable Rotating Stall Cycle - $MG8$

3. Entrained Non-axisymmetric Solutions

Typical parameters $[\Psi_0 \;\; h \;\; w \;\; m \;\; l_c \;\; \mu] = [0.23 \; 0.32 \; 0.18 \; 2.0 \; 4.0 \; 1.0]$ were chosen and the above systems (15) were analyzed numerically in DSTool [5]. In this section we present the results for the three mode model ($n = 3$) and also note the typical trends in the entrained dynamics as the order of the truncation of the disturbance potential is increased. Figure 1 ($top, left$) shows what we will refer to as the primary rotating stall equilibria traced out as the throttle coefficient K_T is varied. The compressor characteristic (4) is shown for reference. Intersections of the throttle characteristic (5) with the entrained stall equilibria correspond to traveling waves where the flow coefficient Φ, the pressure rise coefficient Ψ, the amplitudes of the first through third modes of rotating stall and their phase difference states are *constant*. Recall from Section 2.1, this corresponds to the stall cell rotating around the annulus with fixed shape. Also shown in this figure are the bifurcation diagrams for the other five states including the amplitudes A_1, A_2 and A_3, and phase entrainments $\zeta_1 = \Theta_2 - 2\Theta_1$ and $\zeta_2 = \Theta_3 - 3\Theta_1$ as the throttle coefficient K_T is varied. Stability of these entrained solutions is denoted by either a solid line (stable) or a dashed line (unstable). Figure 2 is a plot of a stable solution ($K_T = 0.45$) versus angular position θ and nondimensional time ξ.

In addition to showing that the lower modes of rotating stall are predominant along this equilibria, from these diagrams we see evidence of a deep hysteresis associated with the primary bifurcation into stall. By deep hysteresis we simply mean that stall equilibria exist for flow coefficients larger than that of the peak of the axisymmetric compressor characteristic. Previously in [6] it was shown that deep hysteresis depends on the *skewness* of the compressor characteristic. A characteristic is said to be left (right) skewed if drops off faster to the right (left) of the peak. A right skewed characteristic is required for stall equilibria to exist to the right of the peak. With a continuous cubic characteristic right skewness is not possible, hence higher order characteristics were required for the model to exhibit deep hysteresis. In this analysis we have assumed a cubic characteristic, hence these bifurcation diagrams show that deep hysteresis is to some extent a multi-mode phenomena.

Upon comparison of the models we have simulated, including truncations of up to five Fourier modes, we see establishment of two trends in this primary rotating stall equilibria as we increase the order of the expansion. The extent of the deep hysteresis increases, as well as the number of relative minimums and maximums in the (Φ, Ψ) plane in the models we have investigated.

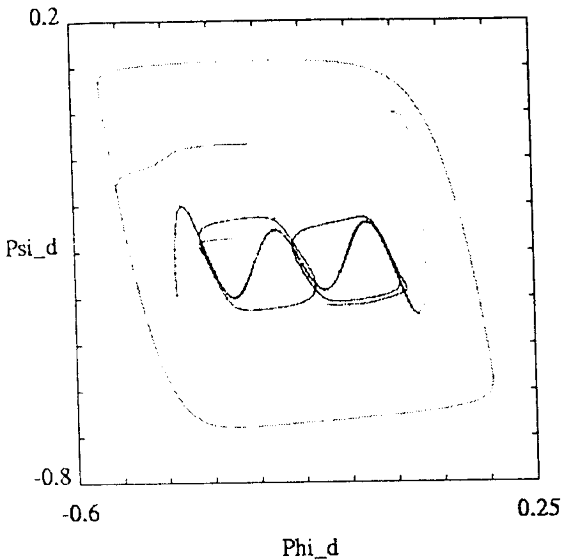

Figure 3: Relaxation Oscillations - $MG8$

3.1. Relaxation Oscillations

The Greitzer surge parameter B was also varied in DSTool, and both axisymmetric and non-axisymmetric relaxation oscillations were found. The typical axisymmetric surge limit cycle exists for large (~ 1.0) B parameters, as shown in Figure 3. Here $\Phi_d = \Phi - \Phi_e$, $\Psi_d = \Psi - \Psi_e$ and (Φ_e, Ψ_e) is the peak of the compressor characteristic. However, in the higher order Galerkin projections of the Moore-Greitzer model investigated in this paper, multiple inner relaxation oscillations were found to form around the relative minimums and maximums of the entrained rotating stall equilibria. Figure 3 shows trajectories that converge to these limit cycles for $K_T = 0.18$ and $K_T = 0.32$. The primary stall equilibria is shown for reference. In Figure 4 we plot the flow coefficient Φ, the pressure rise coefficient Ψ, and the amplitudes of the first, second and third modes of rotating stall (A_1, A_2, and A_3) as functions of nondimensional time ξ for the left trajectory that converges to the inner relaxation oscillation. From these time traces we see that this relaxation oscillation is actually *classic surge*, as stable oscillation occurs simultaneously for the surge and rotating stall states. We emphasize this inner relaxation oscillation is a novel phenomenon not found in the single mode expansion, and this phenomena exists for a much larger range of parameters than the classic surge type behavior in [2].

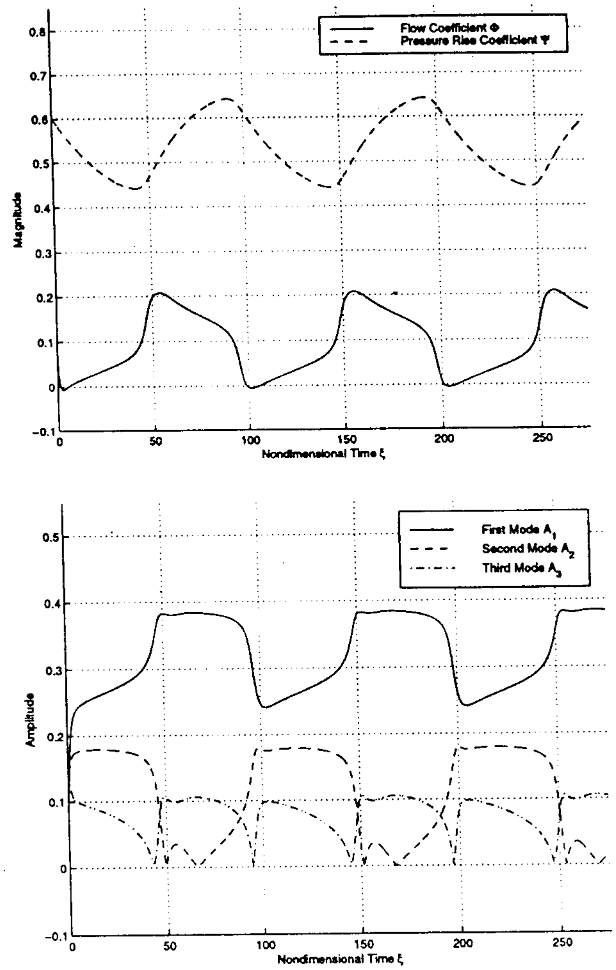

Figure 4: Classic Surge Relaxation Oscillations - $MG8$

4. Elimination of Hysteresis with Feedback in Multi-Mode Models

It was shown previously in [7] that the feedbacks

$$u = K_\Phi(\Phi - \Phi_e) \quad (17)$$
$$u = K_\Psi(\Psi - \Psi_e) \quad (18)$$

each eliminate the hysteresis associated with the primary bifurcation into stall for the single harmonic expansion of the Moore-Greitzer model with a cubic compressor characteristic. Here the subscript e denotes the desired operating point, i.e. the peak of the compressor characteristic. It was also shown in [6] that with higher order right skew compressor characteristics feedback on the stall cell amplitude was required to eliminate hysteresis in the one mode expansion.

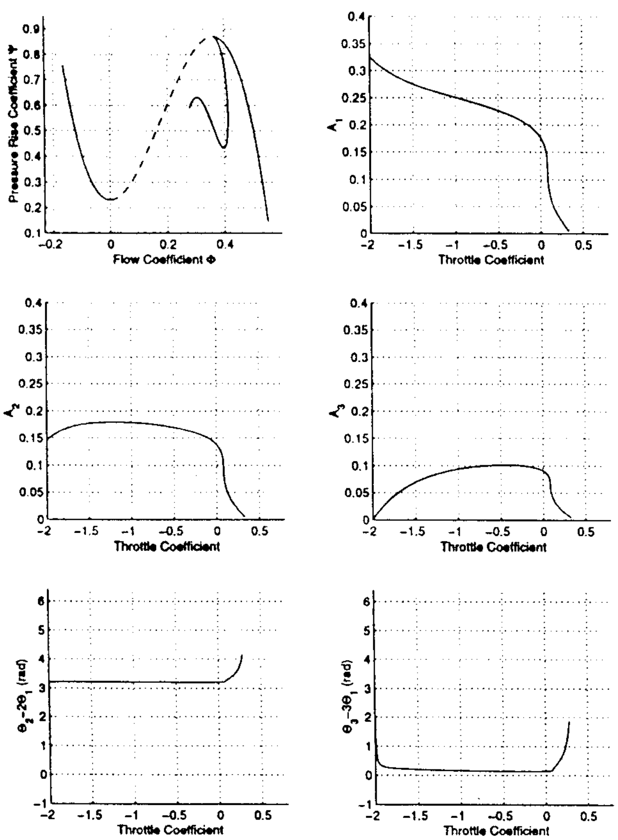

Figure 5: Controlled Bifurcation Diagrams - $MG8$

The behavior of these feedbacks (17,18) in a two mode expansion was investigated in [3]. Stabilization of the peak of the compressor characteristic was achieved, but the global property of hysteresis could not be eliminated. We use the entrainment reduction introduced in this paper to show numerically that in the models we have simulated (up to five modes) that feedback on a combination of the displacement from the peak of the compressor characteristic (17,18) and the absolute value of the amplitude of the first mode eliminates the deep hysteresis associated with the primary bifurcation into stall:

$$u = K_\Phi(\Phi - \Phi_e) + K_\Psi(\Psi - \Psi_e) + K_{A_1}|A_1| \quad (19)$$

Figure 5 shows the results of implementing this feedback in the three mode expansion with gains of $K_\Phi = -2$, $K_\Psi = 2$ and $K_{A_1} = 9$. The top left figure is a plot of the stabilized rotating stall characteristic with the compressor characteristic (4) shown for reference. Also included are the bifurcation diagrams for the modal amplitudes (A_1, A_2, and A_3) and phase entrainment states (ζ_1, ζ_2) as the throttle parameter K_T is varied.

5. Summary

We have discussed the dynamics of higher order Fourier expansions of the Moore-Greitzer model of transients in aeroengine compression systems. With the assumption of steady state entrainment a framework was established which simplified the numerical analysis of non-axisymmetric periodic solutions corresponding to rotating stall. The rotating stall characteristic was shown to exhibit deep hysteresis with a cubic compressor characteristic, establishing the fact that deep hysteresis to a certain extent is a multi-mode phenomena. General trends as the order of the models increased included an increase in the severity of the deep hysteresis and an increase in the number of relative minimums and maximums. New dynamic interactions (relaxation oscillations) of surge and stall representing classic surge were discovered in the higher order models. This framework also facilitated an evaluation of various bleed valve type feedbacks, and elimination of hysteresis associated with the bifurcation into stall was demonstrated in numerical simulations.

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