14.2 $\det M = \det LU = \det L \cdot \det U \leftarrow$ +7

(Graded)

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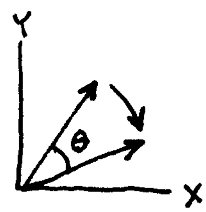
Since both L and U are triangular matrices, their determinants are simply the product of their diagonal elements. In fact, in the <u>standard</u> LU factorization, L is lower <u>unit</u> triangular, so it's diagonal elements are all 1. So $\det L = 1$. i.e. $\det M = \det U$, which is simply the product of the diagonal elements of U. +3

15.1 (i) $L = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

(ii) This is the standard <u>clockwise</u> rotation matrix.



(iii) when $\theta = n\pi$, L will either flip the direction of all vectors, or leave all vectors unchanged. All directions are invariant in these cases, But for any other values of θ, there are <u>NO</u> invariant directions.

(iv) $\begin{vmatrix} \cos\theta - \lambda & \sin\theta \\ -\sin\lambda & \cos\theta - \lambda \end{vmatrix} = \underbrace{\cos^2\theta + \lambda^2 - 2\lambda\cos\theta + \sin^2\theta}_{1} =$

$= \lambda^2 - 2\lambda\cos\theta + 1 := 0$

so $\lambda = \dfrac{2\cos\theta \pm \sqrt{4\cos^2\theta - 4}}{2} = \cos\theta \pm \sqrt{\cos^2\theta - 1} =$

$= \cos\theta \pm \sqrt{-\sin^2\theta} = \cos\theta \pm i\sin\theta.$

(v) We see that λ is only real when $\sin\theta = 0$, i.e. $\theta = n\pi$. In these cases, $\lambda = 1$ (vectors unchanged) or $\lambda = -1$ (vectors flipped).
For any other θ, λ will be complex conjugates.