

Math 129 Spring 2008: Homework 8, Additional Exercise

Let $\Omega > 0$ and $\mathcal{X} = \{f \in L^2 : \hat{f}(w) = 0 \text{ for } |w| > \Omega\}$ a subspace in L^2 .

[A1] Show that the subset $\{s_n(t) = \frac{\sin(\Omega t - n\pi)}{\Omega t - n\pi}\}_{n=-\infty}^{\infty} \subset \mathcal{X}$ forms an orthogonal basis for \mathcal{X} by verifying the following:

- (a) Use Plancherel theorem and the fact that $\{e^{-in\pi w/\Omega}\}_{n=-\infty}^{\infty}$ forms an orthogonal basis for $L^2(-\Omega, \Omega)$ to show that $\{s_n(t)\}_{n=-\infty}^{\infty}$ forms an orthogonal set in \mathcal{X} .
- (b) Show that $\{s_n(t)\}_{n=-\infty}^{\infty}$ is a complete set, i.e., it is a basis for \mathcal{X} .