Practice Midterm 1

Math 145, Spring 2019

Name:

Every solution must contain an explanation written in words supporting your numerical solution to receive credit.

If you need extra space for your solutions, there is an extra page at the back of the exam. If you need extra space for any problem, write CONTINUED IN EXTRA SPACE on the page where the problem is given to you. In the extra space write the problem number that you are solving in that space. **Problem 1:** An hair dresser is planning which days she will cut her clients' hair. She needs to choose 20 out of the 30 days in April to work cutting her clients' hair. She does not want to work more than 1 of the 4 Sundays in April. How many ways are there for her to schedule her working days with clients, with the constraint that she will not work more than one Sunday?

Problem 2: How many numbers from 1 and 100 are a multiple of at least one of the numbers 3, 5, 7?

Problem 3: A new cookie company is giving out 400 cookies for free to people passing by. Each person can take as many cookies as they want (possibly no cookies). At the end of the day, if there are any cookies left, the baker eats the rest. If 100 people pass by during the day, how many different ways are there for the cookies to be distributed?

Problem 4: There are *n* students in class. We know that *k* are in their 1^{st} year at UC Davis and *m* are in their 2^{nd} year at UC Davis. Show that both sides of the following equation give ways of counting the number of ways that the students can be split up into 1^{st} year students, 2^{nd} year students, and students who have been at UC Davis for at least 3 years.

$$\binom{n}{k}\binom{n-k}{m} = \binom{n}{n-k-m}\binom{k+m}{m}$$

Problem 5: Suppose you have a ladder with n rungs. At every step you can go up 1 rung at a time, 2 rungs at a time, or 3 rungs at a time. Define the correct number of initial conditions and a recurrence formula to determine the numbers L_n that count the number of different ways to climb the ladder.