Practice Midterm 2

Math 145, Spring 2019

Name:

Every solution must contain an explanation written in words supporting your numerical solution to receive credit.

If you need extra space for your solutions, there is an extra page at the back of the exam. If you need extra space for any problem, write CONTINUED IN EXTRA SPACE on the page where the problem is given to you. In the extra space write the problem number that you are solving in that space.

Problem 1: How many positive integers are there with *exactly* four digits such that all of the digits are different?

Problem 2: How many subsets of $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ are there which do not contain 2 or do not contain 5?

Problem 3: At a grand opening of a bicycle store, 100 people put their names in a lottery for free bicycle prizes. The store will give out 8 helmets, 5 lights, and 3 locks. How many different outcomes are there for winners of the different bike prizes?

Problem 4: Show that both sides of the following equation count the number of binary strings of 0's and 1's of length n which have at most k 1's where $0 \leq k \leq n-1$:

$$\sum_{j=0}^{k} \binom{n}{j} = \sum_{j=0}^{k} \binom{n-1-j}{k-j} 2^j$$

Problem 5: Let F_n denote the n^{th} Fibonacci number defined by the initial values $F_1 = F_2 = 1$ and the recurrence relation $F_n = F_{n-1} + F_{n-2}$.

Prove that for all $n \geq 1$,

$$-F_1 + F_2 - F_3 + \cdots - F_{2n-1} + F_{2n} = F_{2n-1} - 1$$