## Homework 6

## Math 145, Spring 2019

## Every solution must contain an explanation written in words supporting your numerical solution to receive credit.

1. Find a Hamiltonian cycle (a cycle which includes all the vertices exactly once) in the following graph:



- 2. Given an example of a graph that has a Hamiltonian cycle (a cycle which uses each vertex of the graph exactly once), but has *no* closed Eulerian walk.
- 3. Give an example of a graph which has a Hamiltonian cycle where the Hamiltonian cycle is also a closed Eulerian walk.
- 4. Prove that if G is a tree, and  $v_i$  and  $v_j$  are vertices of G, then there is a *unique* linear subgraph connecting  $v_i$  to  $v_j$ . (i.e. prove that there cannot be two different linear subgraphs connecting  $v_i$  and  $v_j$ )
- 5. Suppose G is a connected graph. Let G' be the graph obtained from G by adding a single edge (no new vertices). Prove that G' contains a cycle.
- 6. Write down all possible trees with at most 5 vertices using the construction procedure from Wednesday's lecture/section 8.2 of the textbook.
- 7. Let T be a tree with vertices of degree only 3 or 1. If T has 10 vertices of degree 3, how many vertices of degree 1 are in T?

8. Find a spanning tree for the following graph:



9. Give the Prüfer code for the following tree (show your work).



10. Give the Prüfer code for the following tree (show your work).

